Relative Velocity Answer Key

Problem 1: Across the river that flows with a rate of 4 m/s, Mr. Pepper swims at the velocity of 3 m/s. Find his velocity relative to the Earth.

Solution: We are given three reference frames: the river (R), Mr. Pepper (M), and the Earth (E). We know that the velocity of the river relative to the Earth $V_{R/E-X}$ = 4 m/s while the velocity of Mr. Pepper relative to the river $V_{M/R-X}$ = 3 m/s.

To find Mr. Pepper's velocity relative to the Earth $V_{M/E-X,}$

$$V_{M/E-X} = V_{M/R-X} + V_{R/E-X}$$

$$V_{M/E-X} = 3 \text{ m/s} + 4 \text{ m/s}$$

$$\mathbf{V_{M/E-X} = 7 \text{ m/s}}$$

For numbers 2 and 3: On a straight two-lane road, you are driving at 40 kph towards the North. On the other lane, a motorcycle approaches you at 56 kph.

Problem 2: Find the motorcycle's velocity relative to you.

Solution: Let us first establish the given reference frames. We have you (Y), the motorcycle (M), and the Earth (E). Let North be our positive x-direction. We are given your x-velocity relative to the Earth, $V_{Y/E-X}$ = 40 kph.

We also know that the motorcycle is approaching you, hence, it is moving south with x-velocity relative to the Earth, $V_{M/E-X}$ = -56 kph. To find the motorcycle's velocity relative to you $V_{M/Y-X}$

$$V_{M/E-X} = V_{M/Y-X} + V_{Y/E-X}$$

$$V_{M/Y-X} = V_{M/E-X} - V_{Y/E-X}$$

$$V_{M/Y-X} = -56 \text{ kph} - 40 \text{ kph}$$

$$\mathbf{V_{M/Y-X} = -96 \text{ kph}}$$

The motorcycle moves at 96 kph in the negative direction relative to you.

Problem 3: Find your speed relative to the motorcycle.

Solution:

$$V_{Y/M-X} = -V_{M/Y-X}$$

$$\mathbf{V_{Y/M-X} = 96\ kph}$$

This means that you are driving at 96 kph in a positive direction relative to the motorcycle.

For numbers 4 and 5: A ship sails North in still water at 25 m/s directly across the sea that runs east at 11 m/s.

Problem 4: What is the velocity of the ship relative to Earth?

Solution: Establishing the given reference frames, we have the ship (H), the sea (S), and the Earth (E). We are given the velocity of the ship relative to the sea, $V_{H/S-X} = 25$ m/s, and the velocity of the sea relative to the Earth, $V_{S/E-X} = 11$ m/s.

To find the velocity of the ship relative to Earth $V_{H/E-X,}$

$$V_{H/E-X} = V_{H/S-X} + V_{S/E-X}$$

The motion of the ship forms a right triangle, so to find the $V_{H/E-X}$, we are going to calculate

$$V_{H/E-X} = \sqrt{(V_{H/S-X})^2 + (V_{S/E-X})^2}$$

$$V_{H/E-X} = \sqrt{(25\ m/s)^2 + (11\ m/s)^2}$$

$$V_{H/E-X} = \sqrt{746\ m^2/s^2}$$

$$\mathbf{V_{H/E-X} = 27.31\ m/s}$$

Problem 5: Find the direction of the ship.

Solution: The direction of the ship can be calculated by

$$\theta = tan^{-1} V_{S/E\text{-}X}/V_{H/S\text{-}X}$$

$$\theta = tan^{-1}\left(\frac{11\ m/s}{25\ m/s}\right)$$

$$\theta = \mathbf{23.75°}$$