Problem 1: If it takes total work W to give an object a speed v and kinetic energy K when starting from rest, find the object's speed (in terms of v) if we do twice as much work on it, again starting from rest.

Solution:

From the equation W = ½m (v_x^2 - v_{0x}^2), we know that $v \; \alpha \; \sqrt{W}$. Hence,

$$\frac{v'}{v} = \frac{\sqrt{2W}}{\sqrt{W}}$$

$$\frac{v'}{v} = \sqrt{2}$$

$$\mathbf{v' = \sqrt{2}\,v}$$

Problem 2: What will be the object's kinetic energy (in terms of K) if we do thrice as much work on it, again starting from rest?

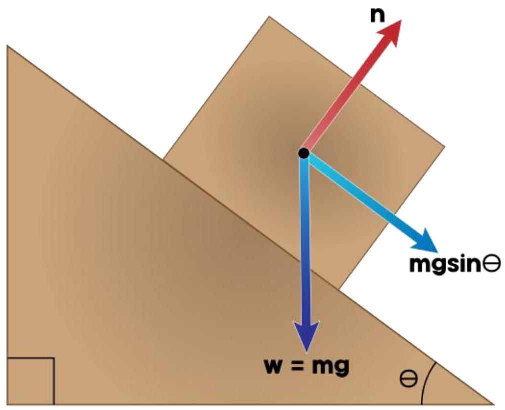
Solution:

From the equation $W = \Delta KE$, we know that $w \; \alpha$ KE. Hence, if we do thrice as much work, the KE will also be thrice as much as before.

Problem 3: Starting from rest, find the final speed of a 3-kg block that slides 2.5 meters along an inclined plane that slopes downward at an angle of 30° below the horizontal. Neglect friction.

Solution:

We know that $W = \Delta KE$. Since the block starts from rest, our v_{0x} = 0, yielding the equation W = ½mv_x^2. Refer to the free body diagram below.



The normal force is perpendicular to the displacement; hence, no work is done. The force along the component of gravity is $F = mg \sin \theta$. Since friction is neglected and we are given with displacement d, W = mgd sin θ.

This gives us

$$mgd \sin \theta = \tfrac{1}{2} m v_x^2$$

$$2gd \sin \theta = v_x^2$$

$$v_x = \sqrt{2gd \sin \theta}$$

$$v_x = \sqrt{2(9.8\ m/s^2)\ (2.5\ m) \sin 30}$$

$$\mathbf{v_x = 5\ m/s}$$

Problem 4: You designed an elevator that carries hollow blocks to ascend 40 m in 35.0 s. It has a mass of 75 kg (does not include the hollow blocks) and its motor can provide up to 3000 watts of power to the elevator. If an average hollow block has a mass of 3 kg, find the maximum number of hollow blocks that can be placed in the elevator.

Solution:

We are given the following quantities:

$h = 40$ m

$t = 35.0$ s

$m_e = 75$ kg

$P = 3000$ watts

$m_{avg} = 3$ kg

We know that P = w/t. The work done against gravity is equal to the total weight of the elevator and the hollow blocks. Hence,

$$P = \frac{mgh}{t}$$

The total mass of the elevator and the hollow blocks is

$$mgd \sin\theta = \tfrac{1}{2} m v_x^2$$

$$2gd \sin\theta = v_x^2$$

$$v_x = \sqrt{2gd \sin\theta}$$

$$v_x = \sqrt{2(9.8\ m/s^2)\ (2.5\ m)\ \sin 30}$$

$$\mathbf{v_x = 5\ m/s}$$

To find the total mass of hollow blocks, we will subtract the mass of the elevator from the total mass calculated.

$$m_{HB} = m_T - m_e$$

$$m_{HB} = 267.9\text{ kg} - 75\text{ kg}$$

$$m_{HB} = 192.9\text{ kg}$$

To find the maximum number of hollow blocks in the elevator, we are going to divide the average mass of the hollow blocks by their total mass.

$$n_{max} = \frac{192.9\ kg}{3\ kg}$$

$$\mathbf{n_{max} = 64\ hollow\ blocks}$$

Problem 5: Find the gravitational potential energy of a 60-kg adventurer who climbs from the 500-m level on a vertical cliff to the top at 1350 m.

Solution: We know that U_{grav} = mgh. Substituting the values, we have

$$U_{grav} = (60\ kg)\ (9.8\ m/s)\ (1350\ m - 500\ m)$$

$$\mathbf{U_{grav} = 499,800\ J}$$