UPCAT Mathematics Answer Key

Set 1

1. Answer: A.

If you think of distance above sea level as a positive number, then you must think of going below sea level as a negative number. Going up is in the positive direction, while going down is in the negative direction. Give all the descending distances a negative sign and the ascending distances a positive sign.

The resulting numerical expression would be as follows: −80 + +25 + −12 + +52

Because addition is commutative, you can associate like-signed numbers: (−80 + −12) + (+25 + +52)

Evaluate the numerical expression in each parentheses: [−80 + −12 = −92] [+25 + +52 = +77]

Substitute the values into the numerical expression: (−92) + (+77)

Signs different? Subtract the value of the numbers and give the result the sign of the higher value number. [92 − 77 = 15]

The diver took his rest stop at −15 feet.

2. Answer: D.

Substitute the values for the variables into the expression.

$(\frac{1}{2})\{(\frac{(6)}{2}-3)-4(3)\}$

Evaluate the expression in the innermost parentheses.

$[(\frac{(6)}{2}-3)=\frac{6}{2}-3]$

PEMDAS: Division before subtraction. Substitute the result into the numerical expression.

$[\frac{6}{2}-3=3-3=0]$

$(\frac{1}{2})\{(0)-4(3)\}$

Evaluate the expression inside the parentheses.

[{0 − 4(3)} = 0 − 4 · 3]

PEMDAS: Multiply before subtraction.

[0 − 4 · 3 = 0 − 12]

Change subtraction to addition and the sign of the term that follows.

[0 − 12 = 0 + −12 = −12]

Substitute the result into the numerical expression

$(\frac{1}{2})\{-12\} = \frac{1}{2} \cdot -12$

Signs different? Multiply numbers and give the result a negative sign.

$[\frac{1}{2} \cdot 12 = 6]$

$\frac{1}{2} \cdot -12 = -6$

The simplified value of the expression is as follows: $y\{(\frac{x}{2} - 3) - 4a\} = -6$

3. Answer: D.

To solve this problem, choose an even integer for x and an odd integer for y and evaluate each of the answer choices. Set x = 0 and y = 1 → only x + y + 2 will evaluate to an odd integer.

4. Answer: A.

35% of 15% of a number is 0.35 * 0.15 of that number:

0.35 * 0.15 = 0.0525x

5. Answer: D.

Translate the given expressions into equations:

x * 0.6 = 12

x = 20

20 * 1.65 = 33

6. Answer: C.

Add 5 to both sides of the inequality. $\frac{4}{3}x - 5 + 5 > x - 2 + 5$

Simplify. $\frac{4}{3}x > x + 3$

Subtract 1x from both sides of the inequality.

$\frac{4}{3}x - x > x - x + 3$

$\frac{4}{3}x - \frac{3}{3}x > x - x + 3$

Simplify the expressions.

$\frac{1}{3}x > 3$

Multiply both sides of the inequality by 3.

$3(\frac{1}{3}x) > 3(3)$

$\frac{3}{1}(\frac{1}{3}x) > 3\ (3)$

Simplify the expressions. $x > 9$

7. Answer: D.

Divide both sides of the equation by 3.

$$\frac{24x^2}{3} = \frac{3(43x - 15)}{3}$$

Simplify terms. $8x^2 = 43x - 15$

Add $(15 - 43x)$ to both sides of the equation. $8x^2 + 15 - 43x = 43x - 15 + 15 - 43x$

Combine like terms. $8x^2 + 15 - 43x = 0$

Use the commutative property to move terms. $8x^2 - 43x + 15 = 0$

Factor the trinomial expression. $(8x - 3)(x - 5) = 0$

Using the zero product property, add 3 to both sides and divide by 8.

$8x - 3 = 0$

$$\frac{8x}{8} = \frac{3}{8}$$

Simplify terms. $x = \frac{3}{8}$

Now let the second factor equal zero. x − 5 = 0

Add 5 to both sides. x = 5

The solutions for the quadratic equation $24x^2 = 3(43x - 15)$ are $x = \frac{3}{8}$ and x = 5.

8. Answer: B.

If the price is no less than 100 Dollars, then the price is either equal to or greater than 100 Dollars.

x ≥ 100

9. Answer: D.

Substitute the coordinates of the given points in the given equation and check which one gives a false statement.

Point (9,- 6) : - 6 = -(9) + 3 , - 6 = - 6 , true , point lies on the line

Point (3,0) : 0 = - (3) + 3 , 0 = 0 , true , point lies on the line

Point (-2,5) : 5 = - (-2) + 3 , 5 = 5 , true , point lies on the line

Point (2,2) : 2 = - (2) + 3 , 2 = 1 , false , point DOES NOT lie on the line

10. Answer: C.

For the relation in C, when x = 2, there are two possible values of y: 3 or 7 and therefore the relation in C is not a function.

11. Answer: D.

We may use the Associative property of multiplication to write

3(x y) = (3 x)y

12. Answer: D.

Solve the inequality

-7x + 6 ≤ -8 , given

-7x + 6 - 6 ≤ -8 - 6 , add - 6 to both sides

-7x ≤ - 14 , simplify

-7x / -7 ≥ -14 / -7 , divide by - 7 and CHANGE symbol of inequality

x ≥ 2 , solution set

The answer to the question is D since 2 is greater that or equal to 2.

13. Answer: A.

We first rewrite the given equation in the form

|-2x - 5| = k + 3

The term |-2x - 5| is either positive or equal to zero. Therefore the above equation has no solutions whenever the expression k + 3 is negative. The values of k for which the above equation has no solutions are solutions of the inequality

k + 3 < 0 or k < - 3

The answer is A since - 5 is less than - 3.

14. Answer: D

I. $\sqrt{x} < x$: if $x = \frac{1}{4}$, $\sqrt{\frac{1}{4}} < \frac{1}{4} \rightarrow \frac{1}{2} < \frac{1}{4}$ (false)

II. $x^2 > x$: if $x = \frac{1}{2}$, $(\frac{1}{2})^2 > \frac{1}{2} \rightarrow \frac{1}{4} > \frac{1}{2}$ (false)

III. $x^2 > \sqrt{x}$: if $x = \frac{1}{4}$, $(\frac{1}{4})^2 > \sqrt{\frac{1}{4}} \rightarrow \frac{1}{16} > \frac{1}{2}$ (false)

None of the above is true.

15. Answer: D.

$4x^2 + 7x + 3 = 2k$

a = 4

b = 7

c = 3 – 2 k

$b^2 – 4ac = 0$

$7^2 – 4(4)(3 - 2k) = 0$

49 – 48 + 32k = 0

k = $-\frac{1}{32}$

16. Answer: A.

m = $\frac{y_2-y_1}{x_2-x_2} = \frac{8-1}{6-0} = \frac{7}{6}$

17. Answer: A.

Here we are given a system of equations and asked to solve in terms of one of the given variables. It is important to note that we are looking for 3b and not solving for b alone. We will begin by solving for c in terms of a using the first equation. We will then substitute this value into the second equation so as to eliminate the c variable and be left with only a's and b's. We will then solve for 3b:

$$\frac{a+4}{3} = c$$

$$\rightarrow b = 2\left(3 - \left(\frac{a+4}{3}\right)\right)$$

$$= 2\left(\frac{9}{3} - \frac{a+4}{3}\right)$$

$$= 2\left(\frac{5-a}{3}\right)$$

Here we will multiply both sides by 3 so we have our target value of 3b on one side, and we will then simplify the expression on the other side:

3b = 2(5 − a)

= 10 − 2a

18. Answer: C.

A. The sum of two odd integers. 15 + 15 = 30, 30 ÷ 10 = 3 (odd integer)

B. The product of two prime numbers. 2 x 5 = 10, 10 ÷ 10 = 1 (odd integer)

C. The product of two odd integers. 3 x 5 = 15, 15 ÷ 10 = 1.5 (**not an integer**)

D. The sum of three consecutive integers. 9 + 10 + 11 = 30, 30 ÷ 10 = 3 (odd integer)

19. Answer: B.

$2^{-|-2|} = 2^{-2}$, since | - 2 | = 2

$= 1 / 2^{2}$, since $a^{-n} = 1 / a^{n}$

= 1/4 = 0.25

20. Answer: B.

Substitute x by -2 in f(x) as follows

$f(-2) = 4(-2)^3 - 4(-2)^2 + 10$

= 4(-8) - 4(4) + 10 = - 32 - 16 + 10 = - 38

21. Answer: D.

The two lines intersect at the point (3 , -4) which is in quadrant IV.

22. Answer: D.

Use the distributive property of multiplication. 3(1) − 3(3x) ≥ −3(x) − 3(27)

Simplify terms. 3 − 9x ≥ −3x − 81

Add 9x to both sides. 3 − 9x + 9x ≥ 9x − 3x − 81

Combine like terms. 3 ≥ 6x − 81

Add 81 to both sides of the inequality. 3 + 81 ≥ 6x − 81 + 81

Combine like terms. $84 \geq 6x$

Divide both sides of the inequality by 6. $\frac{84}{6} \geq \frac{6x}{6}$

Simplify. $14 \geq x$

23. Answer: C.

Multiply the exponents of each factor inside the parentheses by the exponent outside the parentheses.

$3^2x^2y^{10} - 11x^2y^24^2y^8$

Use the commutative property of multiplication.

$3^2x^2y^{10} - 11 \times 4^2x^2y^2y^8$

When similar factors, or bases, are multiplied, add the exponents of the variables.

$3^2x^2y^{10} - 11 \times 16\ x^2y^{10}$

Evaluate numerical factors.

$9x^2y^{10} - 176x^2y^{10}$

Combine like terms.

$-167x^2y^{10}$

24. Answer: A.

This expression can be factored using the trinomial method.

The factors of v4 are $(v^2)(v^2)$, and the factors of 48 are (1)(48) or (2)(24) or (3)(16) or (4)(12) or (6)(8).

Only the product of a positive and a negative numerical term will result in −48. The only factors of 48 that can be added or subtracted in any way to equal 13 are 3 and 16.

Use 3 and 16 and a positive and negative sign in the terms of the trinomial factors. Check your answer using FOIL.

$(v^2 + 3)(v^2 - 16) = v^4 - 16v^2 + 3v^2 - 48 = v^4 - 13v - 48$

You may notice that one of the two factors of the trinomial expression can itself be factored. The second term is the difference of two perfect squares.

Factor ($v^2 - 16$) using the form for factoring the difference of two perfect squares.

$(v + 4)(v - 4) = v^2 - 4v + 4v - 16 = v^2 - 16$

This now makes the complete factorization of $v^4 - 13v^2 - 48 = (v^2 + 3)(v + 4)(v - 4)$.

25. Answer: D.

For this expression, use the product property of radicals and combine the factors in the radicand and outside the radical signs.

$(9\sqrt{a^2 b})(3a\sqrt{b}) = 9 \times 3a\sqrt{a^2 b \times b} = 27a\sqrt{a^2 b^2} = 27a \times ab = 27a^2b$

26. Answer: B.

$16 - x^2 \geq 0$

$x^2 - 16 \leq 0 \rightarrow (x - 4)(x + 4) \leq 0 \rightarrow -4 \leq x \leq 4$

27. Answer: B.

$P = 2L + 2W$

$L = (P - 2W) \div 2$

$= [(16x + 8y) - 2(5x - 2y)] \div 2$

$= [16x + 8y - 10x + 4y] \div 2$

$= [6x + 12y] \div 2$

$L = 3x + 6y$

28. Answer: C.

$\frac{x2}{k} - kx + k = 0 \rightarrow a = \frac{1}{k}$, $b = -k$, $c = k$

It has two real roots if $b^2 - 4ac > 0$

$(-k)^2 - 4(\frac{1}{k})(k) > 0 \rightarrow k^2 - 4 > 0 \rightarrow (k - 2)(k + 2) > 0 \rightarrow$ **k < - 2 or k > 2**

29. Answer: D

The most straightforward method for solving problems of this type is to begin by supplying our own initial value prior to any increase or decrease in profits. Because percents are essentially a division by 100, choosing 100 as our starting value for any percent problem is wise. We are told that for the first year, the percent rose by 12%, so:

100 * 0.12 = 12

So at the start of 2011, the company now has 112, for which there is another increase of 18%, so:

112 * 0.18 = 20.16

Final amount in 2012 = 112 + 20.16 = 132.16

To find the overall percent increase, we find the total increase, then divide this value by the original amount, so:

132.16 − 100 = 32.16

32.16 ÷ 100 = 0.32

= 32%.

30. Answer: D.

$9^{2x+5} = 81^{x+1} \rightarrow 9^{2x+5} = (9^2)^{x+1}$

$\rightarrow 9^{2x+5} = (9)^{2x+2}$

$2x + 5 = 2x + 2$

$5 = 2$ (no solution)

31. Answer: C.

In order to find the average of the set of solutions, we must first solve the inequality:

$-6 < -2x + 4 < 6$

$\rightarrow -10 < -2x < 2$

$\rightarrow 5 > x > -1$

There are a few things to remember when working with absolute values and inequalities. In order to eliminate the absolute value lines, we must be sure that we are considering both the positive and negative distance from 0 in the inequality, an easy way to take care of this is to remove the absolute value lines and treat the expression as if it is also greater than the opposite of the value it is smaller than, for example: −6 < −2x + 4 < 6.

Another important thing to remember is that when multiplying or dividing an inequality by a negative value, we must switch the directions of any/all inequality signs, for example:

$$-10 < -2x < 2$$

$$\rightarrow -\frac{10}{-2} > x > \frac{2}{-2}$$

Now that we have the solution set, we can find its average by summing the extreme values and dividing by 2:

$$\frac{5+(-1)}{2} = \frac{4}{2} = 2$$

32. Answer: D.

To solve this, set x = 100 and perform the operations. One hundred is a great number to choose here because 1% of 100 is 1.

100 − 0.4 * 100 = 60

60 + 0.25 * 60 = 75

Because x = 100 was our initial value, we want to choose the answer choice that gives us 75 when we substitute 100 for x, 0.75 * 100 = 75, so answer choice (D) is correct.

33. Answer: B.

Recall order of operations and the distributive property to answer this question. The second bracket simplifies to z − x + y and to this we multiply (x + y). Begin by distributing the x through (z − x + y) and to this add the distribution of y through (z − x + y) to get the answer.

34. Answer: B.

We can either use the system of equations to solve for x and y individually to find that x = 5 and y = 2 which gives 25 − 4 = 21, or we can see that if we multiply (x + y) and (x − y) we will be left with x2 − y2, 7 * 3 = 21.

35. Answer: A.

To solve this problem we will substitute (x + h) for every x in our function $f(x)$:

$f(x+h) = (x+h)^2 + 2(x+h) + 2$.

Evaluate the expression using the distributive property to arrive at:

$x^2 + 2x + 2xh + 2h + h^2 + 2$.

36. Answer: C.

Recall midpoint formula is the average of the x values and the average of the y values in (x,y) coordinate form.

First find the x coordinate midpoint:

(10 + (−2)) ÷ 2

= 4

Then find the y coordinate midpoint:

(9 + (−4)) ÷ 2

=2.5

Add them together to get the sum:

4 + 2.5 = 6.5

37. Answer: A.

Because we are presented with a pie chart, we know the percentages spent on each department will add up to 100%, meaning the remaining expenditures represent 26% of all expenditures. We can set up a proportion between the 19% spent on R&D and the 26% spent on others. 19/26 = x/12,250. Cross multiply to solve for x to find 8951.9 which rounds to 8952.

38. Answer: A.

Note that $\log_b b^x = x$, $\log_b b^0 = 0$ and $\log_b 1 = 0$

$\log 16^x = \log_8 1 = 0$

$\log 16^x = 0$

$16^x = 10^0 = 1$

$16^x = 16^0$

$x = 0$

39. Answer: A.

Recall that the x-intercept of a function is where the function output $f(x) = 0$. So we begin by setting our function equal to 0 and solving for the target variable x:

$f(x) = 0 = x^3 + 3x^2 - x - 3$

The problem here is that we are most familiar with solving for x when given a linear or a quadratic function and may not easily recall how to factor polynomials of degree 3. However, with some rearranging of terms, we may get a better idea of how to proceed:

$(x^3 + 3x^2) + (-x - 3)$

$\rightarrow x^2(x + 3) - 1(x + 3)$

$\rightarrow (x + 3)(x^2 - 1) = 0$

By grouping our terms together we can find commonalities to factor out to finally arrive at an expression that enables us to directly solve for x. Setting each of the parentheses equal to 0:

$x + 3 = 0, x^2 - 1 = 0$

$\rightarrow x = -3, x = -1, x = 1$

Recall that $x^2 - 1$ is a difference of squares.

Alternatively, if 'a' is a root of a polynomial function, then $f(a) = 0$. We can plug the answer choices into the function to see which answer choice has three values that all yield zero when plugged into the function. Only answer choice (A) works.

40. Answer: B.

$\frac{tan^2\theta - sin^2\theta}{tan^2\theta \ sin^2\theta} = \frac{tan^2\theta}{tan^2\theta \ sin^2\theta} - \frac{sin^2\theta}{tan^2\theta \ sin^2\theta} = \frac{1}{sin^2\theta} - \frac{1}{tan^2\theta} = \csc^2\theta - \cot^2\theta = 1$

41. Answer: C.

Here we are required to initially substitute the known values of a and b into the expressions provided before finding the difference. Substituting the values of a and b into the first expression gives:

(−13)(4) + 3(4)

= −4 + 12

= 8

Substituting the values of a and b into the second expression gives:

(−13)(4) + 3(4)0

= −4 + 3(1)

= −4 + 3

= −1

The difference between the two expressions is 8 − (−1) = 9

Recall that any number (other than 0) to the 0 power is 1 and that subtracting a negative is the same as adding.

42. Answer: B.

Evaluate the expression to find the most simplified form. First evaluate the coefficients:

4 * 2 * 1⁄3 * 1⁄4 = 2⁄3

$x * x^2 = x^3$

$y * y^2 * y^2 = y^5$

$z * z^3 * z = z^5$

Combine each of these terms to get answer choice (B).

43. Answer: C.

| 2(x – 1) – 15 | = 7

| 2x – 2 – 15 | = 7

| 2x – 17 | = 7

2x – 17 = – 7 and 2x – 17 = 7

2x = 10 and 2x = 24

x = 5 and x = 12

The solution set: { 5,12 }

44. Answer: C.

For this question, you have to examine all the answer options individually in order to eliminate all those that cannot be true. First, if x is positive and y is negative, their product must be negative, so (A) is incorrect.

Next, the sum of a positive and a negative number could be either positive or negative, depending on which number has the greater absolute value; this rules out (B) because it's not always true.

Similar reasoning applies to choice (D) as well. However, both positive and negative real numbers have positive squares, and adding those positive squares will always yield a positive number, so (C) is correct.

45. Answer: C.

From the chart, we can see that Store X had 80 (thousand) in profits and Store Z had 100 (thousand) in profits. Combining these two, we arrive at 180 (thousand) in profits.

46. Answer: B.

We solve this problem by replacing every x in h(x) with 2x − 3 and evaluating the expression:

h(2x − 3) = 3(2x − 3) + 4

= 6x − 9 + 4

= 6x – 5

47. Answer: A

Let a = -0.9 and b = -0.1

A. b – a = (-0.1) – (- 0.9) = 0.8

B. a + b = (-0.1) + (-0.9) = -1.0

C. a – b = (-0.9) – (-0.1) = -0.8

D. 2b – a = 2(-0.1) – (-0.9) = 0.7