UPCAT Mathematics Answer Key

Set 3

1. Answer: B.

Let x = the number of months. The number of months (x), times 12 (pounds per month), plus the starting weight (20), will be equal to 200 pounds.

An equation that represents these words would be 12x + 20 = 200.

Subtract 20 from both sides of the equation. 12x + 20 − 20 = 200 − 20

Associate like terms. 12x + (20 − 20) = 200 − 20

Perform numerical operations. 12x + (0) = 180

Divide both sides of the equation by 12.

$$\frac{12x}{12} = \frac{180}{12}$$

x = 15

The farmer would have to wait 15 months before selling his hog.

2. Answer: A.

Method 1: Quick Method:

The number of girls participating is 105. It was stated in the problem that there are twenty-five more girls than the number of boys that participate. This means that the number of boys that participate is twenty-five less than the number of girls that participate.

Thus, we can simply subtract 25 from the number of girls that participate (which is 105) to obtain the number of boys that participate:

105-25=80

Hence, **the number of boys that participate is 80.**

Method 2: Algebraic Method:

Let *b* be the number of boys that is participate.

Since there are twenty-five more girls than the number of boys, we can express the number of girls that participate as: *b* + 25

Since the number of girls participating is 105, we can equate *b* + 25 to 105:

b+25=105

We are now going to solve for *b* in the equation to find the number of boys that participate.

Subtracting both sides of the equation by 25:

b=80

Hence, **the number of boys that participate is 80.**

3. Answer: C.

The slope of the given (in slope intercept form) line is equal to - 5. Let m be the slope of the line perpendicular to the given line. Two lines are perpendicular if the product of their slopes is equal to -1. Hence

m*(-5) = - 1 Solve for m. Hence

m = 1/5 is the slope of a line perpendicular to the given line.

4. Answer: A.

The three denominators are different and therefore we need to find a common denominator.

We first find the lowest common multiple (LCM) of the two denominators 8, 12 and 16.

8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80,...

12: 12, 24, 36, 48, 60, 72, 84, 96,...

16: 16, 32, 48, 64, 80, 96...

The lowest common denominator is 48 and we now convert all 3 denominators to the common denominator 48 and simplify as follows:

$$\frac{5}{8}+\frac{1}{2}-\frac{5}{16}=\frac{5\cdot 6}{8\cdot 6}+\frac{1\cdot 4}{12\cdot 4}-\frac{5\cdot 3}{16\cdot 3}=\frac{30}{48}+\frac{4}{48}-\frac{15}{48}=\frac{30+4-15}{48}=\frac{19}{48}$$

5. Answer: D.

The two rational expressions have different denominators. In order to add the rational expressions above, we need to convert them to a common denominator. We factor completely the two denominators $x^2 + 6x + 5$ and $x^2 + 11x + 30$ and find the LCD.

$x^2 + 6x + 5 = (x + 1)(x + 5)$

$x^2 + 11x + 30 = (x + 6)(x + 5)$

LCD = (x + 1)(x + 5)(x + 6)

We also factor the numerators.

$x^2 + 3x + 2 = (x + 1)(x + 2)$

$x^2 + 4x - 12 = (x + 6)(x - 2)$

We rewrite the given expression with numerators and denominators in factored form and simplify if possible.

$$\frac{x^2+3x+2}{x^2+6x+5} - \frac{x^2+4x-12}{x^2+11x+30} = \frac{(x+1)(x+2)}{(x+1)(x+5)} - \frac{(x+6)(x-2)}{(x+6)(x+5)}$$

$$= \frac{\cancel{(x+1)}(x+2)}{\cancel{(x+1)}(x+5)} - \frac{\cancel{(x+6)}(x-2)}{\cancel{(x+6)}(x+5)} = \frac{x+2}{x+5} - \frac{x-2}{x+5}$$

$$= \frac{x+2-(x-2)}{x+5} = \frac{4}{x+5}$$

6. Answer: D.

Apply the multiplication rule.

$$\frac{2x+4}{x-5} \cdot \frac{3x-15}{x+2} = \frac{(2x+4)(3x-15)}{(x-5)(x+2)}$$

Factor if possible

$$= \frac{2(x+2)3(x-5)}{(x-5)(x+2)}$$

Simplify if possible

$$= \frac{2\cancel{(x+2)}3\cancel{(x-5)}}{\cancel{(x-5)}\cancel{(x+2)}} = 6 \text{ for } x \neq -2 \text{ and } x \neq 5$$

7. Answer: C.

We first convert (x - 2) into a rational expression. Hence

$$\frac{-2x+4}{x-1} \div (x-2) = \frac{-2x+4}{x-1} \div \frac{x-2}{1}$$

The division of two rational expressions is done by multiplying the first by the reciprocal of the second as follows (see divison rule above).Hence

$$= \frac{-2x+4}{x-1} \cdot \frac{1}{x-2}$$

Multiply numerators and denominators (multiplication rule) but do not expand.

$$= \frac{(-2x+4) \cdot 1}{(x-1)(x-2)}$$

Factor the terms - 2 x + 4 included in the numerator as follows:

- 2 x + 4 = -2(x - 2)

and use - 2 x + 4 in factored form in the rational expression to simplify

$$= \frac{-2(x-2) \cdot 1}{(x-1)(x-2)} = \frac{-2\cancel{(x-2)} \cdot 1}{(x-1)\cancel{(x-2)}} = \frac{-2}{x-1} \quad \text{for} \quad x \neq 2$$

8. Answer: C.

Because we are provided only the ratio of boys to girls, we can provide our own number of boys and girls so long as it satisfies the provided ratio. We will assume there are 10 girls, which means there are 20 boys. Now, ¾ of the boys would be 15 and ½ of the girls would be 5. The ratio of 15:5 is 3:1.

9. Answer: A.

1st: 7:00 A.M., Sunday

2nd: 2:00 P.M., Sunday

3rd: 9:00 P.M., Sunday

4th: 4:00 A.M., Monday

5th: 11:00 A.M., Monday

6th: 6:00 P.M., Monday

10. Answer: A.

Red = x
Green = 3x
White = 2(3x) = 6x
Total: 10x

P(white) = $\frac{6x}{10x} = \frac{3}{5}$

11. Answer: C.

The first term a_1 is 2 and the common difference is equal to: 5 - 2 = 8 - 5 = 3

Hence using the formula for the nth term, $a_n = a_1 + (n - 1)d$ to the term equal to 227, we can write the equation:

227 = 2 + (n - 1)3

Solve the above for n

n - 1 = (227 - 2) / 3 = 75 and n = 76

The 76th term is equal to 227.

12. Answer: D.

The first term $a_1 = 9$ and d = 2 (the difference between any two consecutive odd integers). Hence the sum S_n of the n terms may be written as follows

$S_n = (n/2)[2*a_1 + (n - 1)d] = 15,960$

With $a_1 = 9$ and d=2, the above equation in n may be written as follows

$n_2 + 8\ n - 15860 = 0$

Solve the above for n

n = 122 and n = -130

The solution to the problem is that 122 consecutive odd numbers must be added in order to obtain a sum of 15,860.

13. Answer: A

A probability is always greater than or equal to 0 and less than or equal to 1, hence only A above cannot represent probabilities.

14. Answer: C.

We construct a table of frequencies for the the blood groups as follows

group	frequency
a	50
B	65
O	70
AB	15

We use the empirical formula of the probability

$$P(E) = \frac{\text{Frequency for O blood}}{\text{Total frequencies}}$$

= 70 / 200 = 0.35

15. Answer: C.

The probability of selecting a red marble on the first draw is 10⁄32 because there are 10 red marbles and 32 total marbles. After removing the first red marble there are now 9 red marbles and 31 total marbles left so 9⁄31 chance of selecting the second red marble. To find the probability of both events occurring, we multiply the probabilities to get 9 * 10⁄32 * 31 which reduces to 45⁄496.

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16. Answer: C.

Recall that when provided with a point and the slope of a line, we can use point-slope formula to write an equation for the line. The point slope formula is y − y1 = m(x − x1) where (x1, y1) is the point provided and m is the slope. Plug in the point and slope provided and solve for y:

y − 3 = −⅓ (x − 2)

y = −⅓x + $\frac{11}{3}$

17. Answer: D.

In approaching this problem, consider the number of options the students have for each role. As a role is taken up, there is 1 less student to fill the next role. For President there are 6 options, for Vice President 5 options, for Secretary 4 options, and for Treasurer 3 options. Multiply each of these to find 360 different groups:
6 * 5 * 4 * 3 = 360

18. Answer: C.

Begin by rounding the number to the nearest hundredth: 89.88. Now add the tenths place, 8, and the hundredths, 8, to get 16.

19. Answer: C.

Use an equation to represent the situation, x + x + 2 + x + 4 + x + 6 = 36. Solve for x to find 6, but recognize that this is the smallest integer in the set, the 3rd largest would be 10.

20. Answer: C.

Find n:
$a_n = a_1 + (n - 1)d$
-29 = 91 + (n-1)(-6)
6(n – 1) = 91 + 29
n – 1 = 120 ÷ 6
n = 21

S = sum

= n (a_1 + a_n) ÷ 2
= 21 (91 + [- 29]) ÷ 2
= 21 (62) ÷ 2
= 21 (31)
S = 651

21. Answer: D.

Since in this case the number of scores is even, the median is the average of the two middlemost scores.

median = $\frac{50+51}{2} = \frac{101}{2} = 50.5$

22. Answer: C.

The number of ways of arranging **n** objects in a round table is (n – 1)! Ways. For the five students the number of arrangements is (5 – 1)! = 4! = 24

23. Answer: B.

Solve this problem by setting up a proportion. We are told the ratio of milk to juice is 13:x and that there are 39 milk and 18 juice so → 13/x = 39/18, cross multiply and solve for x to get 6.

24. Answer: C.

$$\frac{\boldsymbol{P_1}}{\boldsymbol{P_2}} = \sqrt{\frac{\boldsymbol{A_1}}{\boldsymbol{A_2}}} = \sqrt{\frac{\boldsymbol{64}}{\boldsymbol{729}}} = \frac{\boldsymbol{8}}{\boldsymbol{27}} = \boldsymbol{8:27}$$

25. Answer: C.

$$\frac{a+b+c}{3} = \frac{a+b}{2} \rightarrow 2(a+b+c) = 3(a+b) \rightarrow 2a+2b+2c = 3a+3b \rightarrow 2c = a+b \rightarrow c = \frac{a+b}{2}$$

26. Answer: B.

Let n be the number of tickets.

$$n = 4 + \frac{(n-4)}{2} + 12 + 15 \rightarrow n = \frac{(n-4)}{2} + 31 \rightarrow 2n = n - 4 + 62 \rightarrow n = 58$$

27. Answer: B.

Let x = original number of candies

Catherine: $\frac{1}{6}x$

Farah: $\frac{2}{5}x$

Wendy: 4

Jane: 100

x = $\frac{1}{6}x + \frac{2}{5}x$ + 4 + 100

x = $\frac{1}{6}x + \frac{2}{5}x$ + 104

30x = 5x + 12x + 3120

13x = 3120

x = 240

28. Answer: C.

Begin by setting up an equation representing the average. (2 + x + 31) ÷ 7 = 24. Solve for x to find 135 and recognize that this x represents the sum of the remaining 5 scores. To find the average, divide 135 by 5 to find 27.

29. Answer: B.

Recall that vertex- form of a parabola is:

$a(x - h)^2 + k$, where (h, k) represents the vertex.

We wish to translate our vertex from (0,0) to (4,−6) so h = 4 and k = −6.

$f(x) = (x - 4)^2 - 6$

30. Answer: B.

Recall that slope-intercept form is y = mx + b where m is the slope and b is the y-intercept. Solve for y:

$8x - 2y = -6$

$2y = 8x + 6$

Divide everything by 2:

$y = 4x + 3$

31. Answer: B.

If the product of two numbers is positive, the two numbers must have the same sign. That is, if ab > 0, then either a > 0 and b > 0, or a < 0 and b < 0.

We are told that A < −1 (which implies that A < 0).

So we know that B < 0.

We also know that AB = 1, so A = 1/B

Since A = 1/B, and A < -1, we can infer that 1/B < -1

If we take reciprocals on both sides of the last inequality, we must flip the inequality sign. Hence: $B > -1$

So we know that $B < 0$, and $B > -1$. We can represent this as a compound inequality: $-1 < B < 0$

32. Answer: C.

Because both of these equations are already solved for the variable x, we can set them equal to each other to find the value of y. Begin by multiplying both sides by 3 to remove the denominator.

$y - 7 = y + 4$

Notice that this equation will never be true. Since there is no solution, so we can conclude that the lines do not intersect.

33. Answer: B.

Recall the slope-intercept form of a line:

$y = mx + b$ where m is the slope.

Solve the given equation for y to find the slope:

$2x - 6 - 6y = 10$

$-6y = -2x + 16$

$y = 1/3x - 16/6$

Slope is equal to 1/3.

34. Answer: D.

She runs for 20 minutes and arrived 5 minutes late → She needs to be exactly there in 15 minutes.

Using a bike with a speed of 1/3 km per minutes → t = d/r → t = 2/1/3 → t = 6

15 minutes – 6 minutes = 9 minutes earlier.

35. Answer: C.

Solve the inequality for one variable:

$y + 3 > -3x + 6$

$y > -3x + 3$

This states that the y-coordinate must be larger than −3 times the x-coordinate plus 3. Test the points provided to see which one satisfies the given inequality (this can also be done graphically). Only (−3, 15) satisfies the inequality.