The four fundamental operations are the heart of arithmetic.

If we can perform these mathematical operations with integers, we can also perform them on fractions and decimals. A lot of real-life world problems that cannot be addressed by whole numbers can be answered using the operations on these rational numbers.

In this reviewer, you'll get the most comprehensive guide to performing the four fundamental operations on fractions and decimals. Most importantly, you'll also learn how they can be applied to solve real-life word problems.

Part I: Operations on Fractions.

Addition and Subtraction of Similar Fractions.

As you can recall, similar fractions are fractions that have the same denominator. The rules on adding and subtracting similar fractions are the same. To add or subtract similar fractions, follow these steps:

1. Add or subtract the numerators of the given fractions and use the sum or difference as the numerator of the resulting fraction.
2. Copy the denominator of the given fractions and use it as the denominator of the resulting fraction.
3. Reduce the answer to its lowest terms, if possible.

To summarize: In order to add or subtract similar fractions, you first need to add or subtract the numerator, then copy the denominator. Afterward, simplify your answer to its lowest terms.

Example 1: $3/5 + 1/5$.

Solution:

Step 1: *Add the numerators of the given fractions and use the sum or difference as the numerator of the resulting fraction.*

$$\frac{3}{5} + \frac{1}{5} = \frac{4}{}$$

We just add the numerators of the given fractions which are 3 and 1 and put the answer as the numerator of the resulting fraction.

Step 2: *Copy the denominator of the given fractions and use it as the denominator of the resulting fraction.*

$$\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$$

The denominator of the given fractions is 5. Hence, we will use 5 as the denominator of the common fraction.

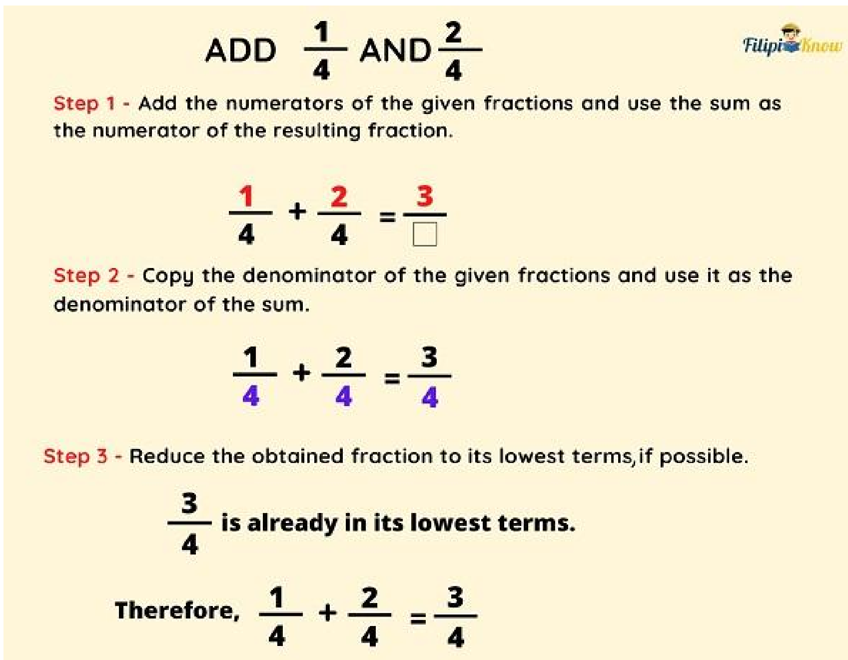
Step 3: *Reduce the answer to its lowest terms, if possible.*

$4/5$ is a fraction in the lowest terms. Hence, no need to simplify it further. Therefore, our final answer is $4/5$.

Let's try to answer more examples:

Example 2: Add 1⁄4 and 2⁄4.

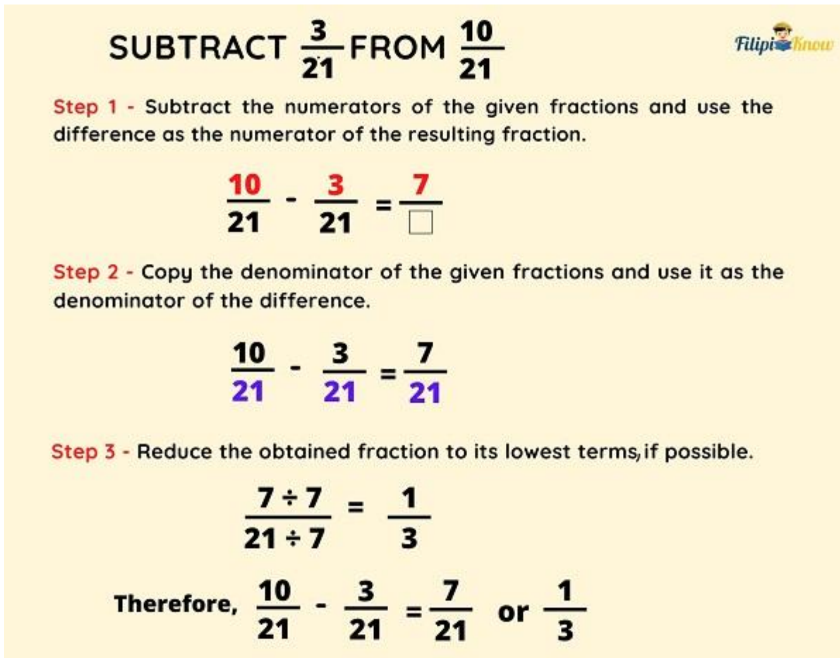
Solution:



Example 3: Subtract 3⁄21 from 10⁄21.

Solution:

Note that the problem asked us to subtract 3⁄21 from 10⁄21. This means that the minuend (the number being subtracted from) is 10⁄21 while the subtrahend (the number being subtracted) is 3⁄21.



Example 4: Bea ate 2⁄8 of the pie that her mother prepared. Meanwhile, Bea's brother ate 4⁄8 of the same pie that Bea ate. What is the total fraction of the pie eaten by Bea and her brother?

Solution:

We can answer this question by adding the fraction of the pie eaten by Bea and the fraction of the pie eaten by her brother. Since 2⁄8 and 4⁄8 are similar fractions, we can use the steps we have for adding similar fractions.

Step 1: *Add the numerators of the given fractions and use the sum as the numerator of the resulting fraction*.

$$\frac{2}{8} + \frac{4}{8} = \frac{6}{\ }$$

Step 2: *Copy the denominator of the given fractions and use it as the denominator of the resulting fraction.*

$$\frac{2}{8} + \frac{4}{8} = \frac{6}{8}$$

Step 3: *Reduce the answer to its lowest terms, if possible.*

6⁄8 is not in its lowest terms yet since 6 and 8 have a Greatest Common Factor (GCF) of 2. Hence, we divide both 6 and 8 by 2.

$$\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

Therefore, Bea and her brother ate 6⁄8 or ¾ of the pie.

Transforming Dissimilar Fractions to Similar Fractions.

Do you still remember what dissimilar fractions are? These are fractions with different denominators. Before you can add or subtract dissimilar fractions, you should transform them first as similar fractions. *But how is that possible?*

These are the steps on how to transform dissimilar fractions into similar fractions:

1. Find the Least Common Multiple (LCM) of the denominators. The number that you will obtain is the Least Common Denominator (LCD). Use the LCD as the new denominator of the fractions.
2. Divide the LCM you have obtained by the denominator of the first fraction. Multiply the resulting number by the numerator. The number that you will obtain is the numerator for the new fraction.
3. Apply Step 2 for the second fraction.

Example: Transform the fractions 3⁄5 and 1⁄3 into similar fractions.

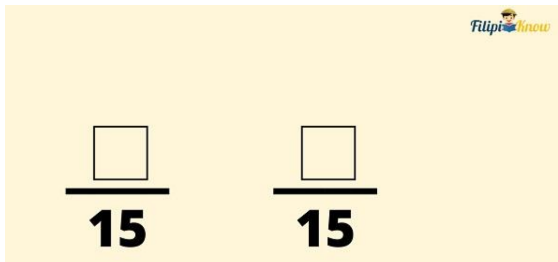
Solution:

Let us apply all the steps previously discussed.

Step 1: *Find the Least Common Multiple (LCM) of the denominators. The number that you will obtain is the Least Common Denominator (LCD). Use the LCD as the new denominator of the fractions.*

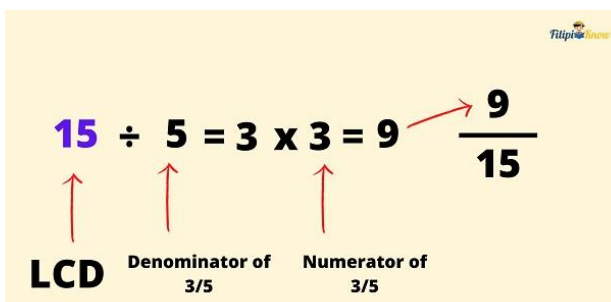
The Least Common Multiple of 5 and 3 is 15 (we colored it with purple in the list). 15 will be our Least Common Denominator (LCD).

We will use 15 as the denominator of our fractions. We leave the numerators of the fractions blank because we need to compute them in the next step.



Step 2: *Divide the LCM you have obtained by the denominator of the first fraction. Multiply the resulting number by the numerator. The number that you will obtain is the numerator for the new fraction.*

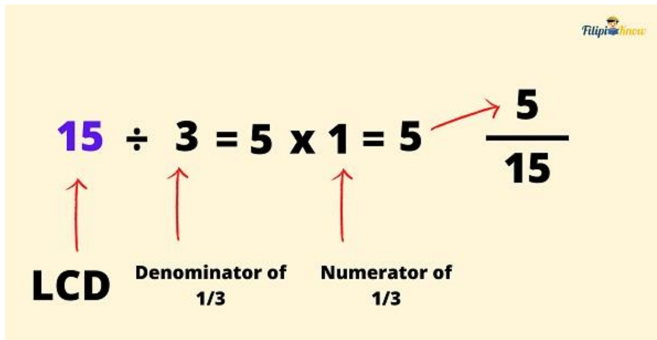
Let us apply this step to 3⁄5.



The LCD we have obtained is 15. We divide the LCD by the denominator of ⅗. Thus, 15 ÷ 5 = 3. Afterward, we multiply 3 to the numerator of ⅗. Hence, 3 x 3 = 9. Therefore, the new numerator is 9.

Step 3: *Apply Step 2 for the second fraction*.

We will do the same thing we performed on ⅗ for the second fraction which is ⅓. We divide the LCD of 15 by the denominator of ⅓ which is 3. Thus, 15 ÷ 3 = 5. Afterward, we multiply 5 by the numerator of ⅓. Hence, 5 x 1 = 5. The new numerator for the second fraction is 5.



When we transform the fractions 3⁄5 and 1⁄3 into similar fractions, we have 9⁄15 and 5⁄15

Transforming dissimilar fractions into similar fractions is an important step in adding and subtracting dissimilar fractions. This means you should master the method presented above before proceeding to the next section of this reviewer.

Addition and Subtraction of Dissimilar Fractions.

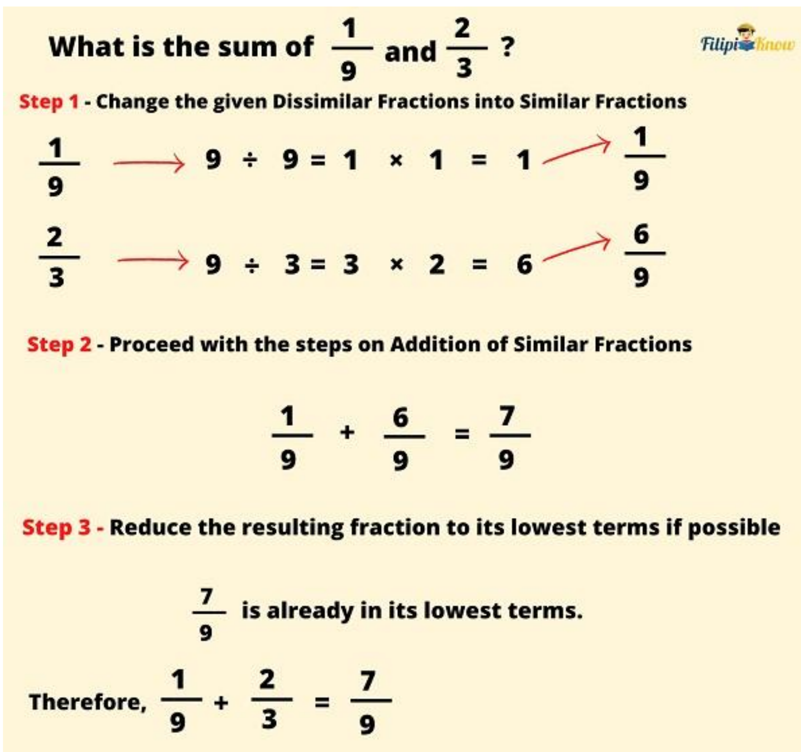
Here are the steps on how to add or subtract dissimilar fractions:

1. Change the given dissimilar fractions into similar fractions (refer to the section above for the steps on transforming dissimilar fractions to similar fractions).
2. Proceed with the steps on the addition or subtraction of similar fractions.
3. Reduce the resulting fraction to its lowest terms, if possible.

Let us try the steps above for the examples below.

Example 1: What is the sum of 1⁄9 and 2⁄3?

Solution:

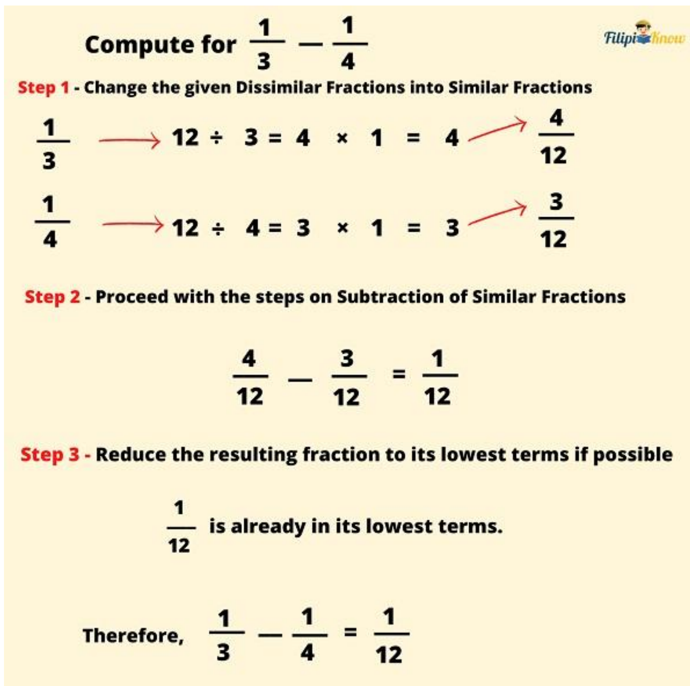


The LCD of 3 and 9 is 9. Hence, we used it as the new denominator of our fractions. Afterward, we performed the steps on changing dissimilar fractions into similar fractions. In Step 2, we just added the numerators: 1 + 6 = 7 and copied the denominator 9. Thus, we obtained a fraction of 7⁄9.

7⁄9 is already in its lowest terms so no need to simplify it further. Hence, the final answer is 7⁄9.

Example 2: Compute for 1⁄3 - 1⁄4.

Solution:



The LCD of 3 and 4 is 12. Thus, we used it as the new denominator of the fractions. Afterward, we applied the steps on transforming dissimilar fractions to similar fractions. Thus, we obtained 4⁄12 and 3⁄12. In Step 2, we just subtracted the numerators: 4 - 3 = 1 and then copied the denominator of 12. Thus, we obtained a fraction of 1⁄12.

Since 1⁄12 is already in its lowest terms, there is no need to simplify it further. Therefore, the final answer is 1⁄12.

Addition and Subtraction of Mixed Numbers.

You are now familiar with adding and subtracting similar or dissimilar fractions. *How about mixed numbers or those combinations of a whole number and a proper fraction? Can we also add or subtract them?* Of course, we can.

Here are the steps you need to follow if you are adding or subtracting mixed numbers:

1. Add or subtract the whole numbers. The resulting number is the whole number part of the sum or difference.
2. Add or subtract the proper fractions. If the given fractions are similar fractions, just add or subtract the numerators then copy the denominator. If the given fractions are dissimilar fractions, make the fractions similar first.
3. Combine the whole number you obtained from Step 1 and the proper fraction you obtained from Step 2 to arrive at a mixed number.
4. Reduce the proper fraction to its lowest terms, if possible.

Example: Add 1 1⁄3 and 4 2⁄5.

Solution:

Step 1: *Add the whole numbers. The resulting number is the whole number part of the sum.*

The whole number parts of 1 1⁄3 and 4 2⁄5 are 1 and 4, respectively. Adding the whole numbers:

$$1 + 4 = 5$$

Therefore, 5 is the whole number part of our sum.

Step 2: *Add the proper fractions. If the given fractions are similar fractions, just add the numerators then copy the denominator. If the given fractions are dissimilar fractions, make the fractions similar first.*

The proper fractions of 1 1⁄3 and 4 2⁄5 are ⅓ and ⅖, respectively. These proper fractions are dissimilar fractions so we need to transform them first into similar fractions.

If we transform ⅓ and ⅖ into similar fractions, we will have (refer to our previous section to review how to transform dissimilar fractions to similar fractions):

1⁄3 → 5⁄15

2⁄5 → 6⁄15

Now, we add the similar fractions:

5⁄15 + 6⁄15 = 11⁄15

Step 3: *Combine the whole number you obtained from Step 1 and the proper fraction you obtained from Step 2 to arrive at a mixed number.*

The whole number that we have obtained from Step 1 is 5. Meanwhile, the proper fraction we have obtained from Step 2 is 11⁄15. Combining them, we have 5 11⁄15.

Step 4: *Reduce the proper fraction to its lowest terms, if possible.*

Since 11⁄15 is in its lowest terms, then we do not need to simplify it.

Therefore, 1 1⁄3 + 4 2⁄5 = 5 11⁄15.

Multiplication of Fractions.

Multiplying fractions are a lot easier than adding or subtracting fractions because you do not have to consider whether the fractions are similar or dissimilar. To multiply fractions, all you have to do is follow these three steps:

1. Multiply the numerators of the given fractions. The resulting number is the numerator of the product (or answer).
2. Multiply the denominators of the given fractions. The resulting number is the denominator of the product (or answer).
3. Reduce the product (or answer) to its lowest terms, if possible.

We can summarize these three steps this way: Multiply numerator by numerator and then denominator by denominator. Afterward, reduce the product to its lowest terms.

Example 1: Multiply 3⁄4 by 1⁄5.

Solution:

Step 1: *Multiply the numerators of the given fractions. The resulting number is the numerator of the product (or answer).*

$$\frac{3}{4} \times \frac{1}{5} = \frac{3}{\quad}$$

The numerators of the given fractions are 3 and 1. When we multiply them, we will obtain 3 x 1 = 3. Hence, 3 is the numerator of our resulting fraction.

Step 2: *Multiply the denominators of the given fractions. The resulting number is the denominator of the product (or answer).*

$$\frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$$

Step 3: *Reduce the product (or answer) to its lowest terms, if possible.*

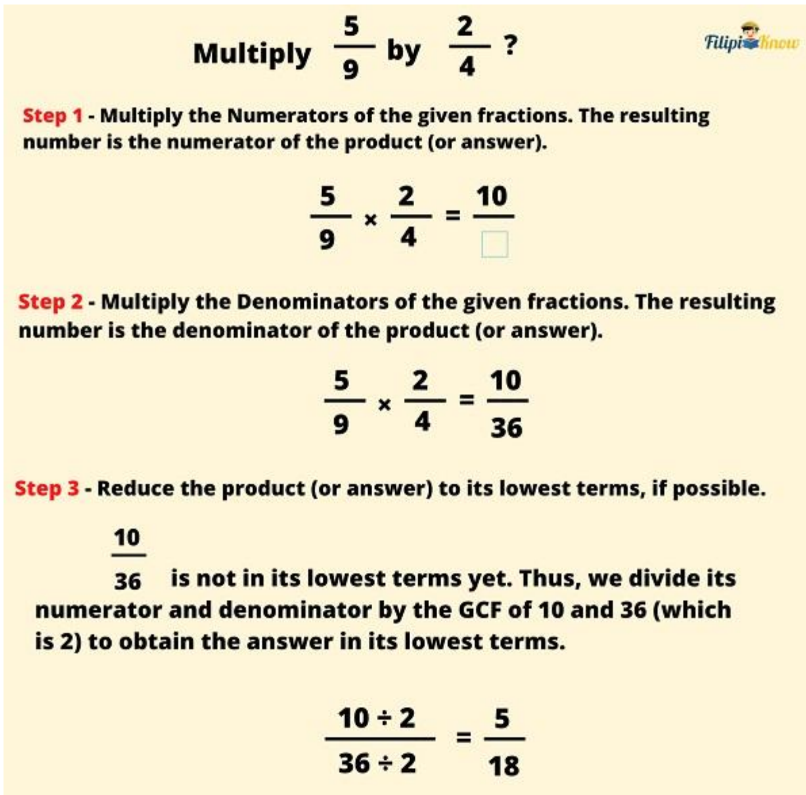
3⁄20 is a fraction that is already in the lowest terms. Hence, no need to simplify it further.

Therefore, our final answer is 3⁄20.

Let us have more examples:

Example 2: Multiply 5⁄9 by 2⁄4.

Solution:



Therefore, the product is 5⁄18.

Example 3: What is 2⁄5 of 50?

Solution:

The word "of" is actually a signal word for the multiplication of fractions. Hence, the question above can be interpreted also as 2⁄5 × 50.

But, *how do we multiply a fraction by a whole number or vice versa?*

The answer is simple! Just put a denominator of 1 for the whole number:

$$\frac{2}{5} \times \frac{50}{1}$$

Afterward, proceed with the steps on multiplying fractions.

$$2/5 \times 50/1 = 100/5$$

Note that we can simplify 100⁄5 as 20⁄1.

If the denominator of a fraction is 1, it means that the fraction is equal to the whole number indicated in the numerator.

Therefore, 20⁄1 = 20

Hence, 2⁄5 of 50 is equal to 20.

Example 4: What is 3⁄4 of 100?

Solution:

This question can be solved using the same method we used for the previous example. Again, the word “of” is a signal word for the multiplication of fractions.

Let us start by putting a denominator of 1 for 100:

$$3/4 \times 100/1$$

Multiply the numerators as well as the denominators:

$$3/4 \times 100/1 = 300/4$$

We can simplify 300⁄4 as 75⁄1 which is equal to 75.

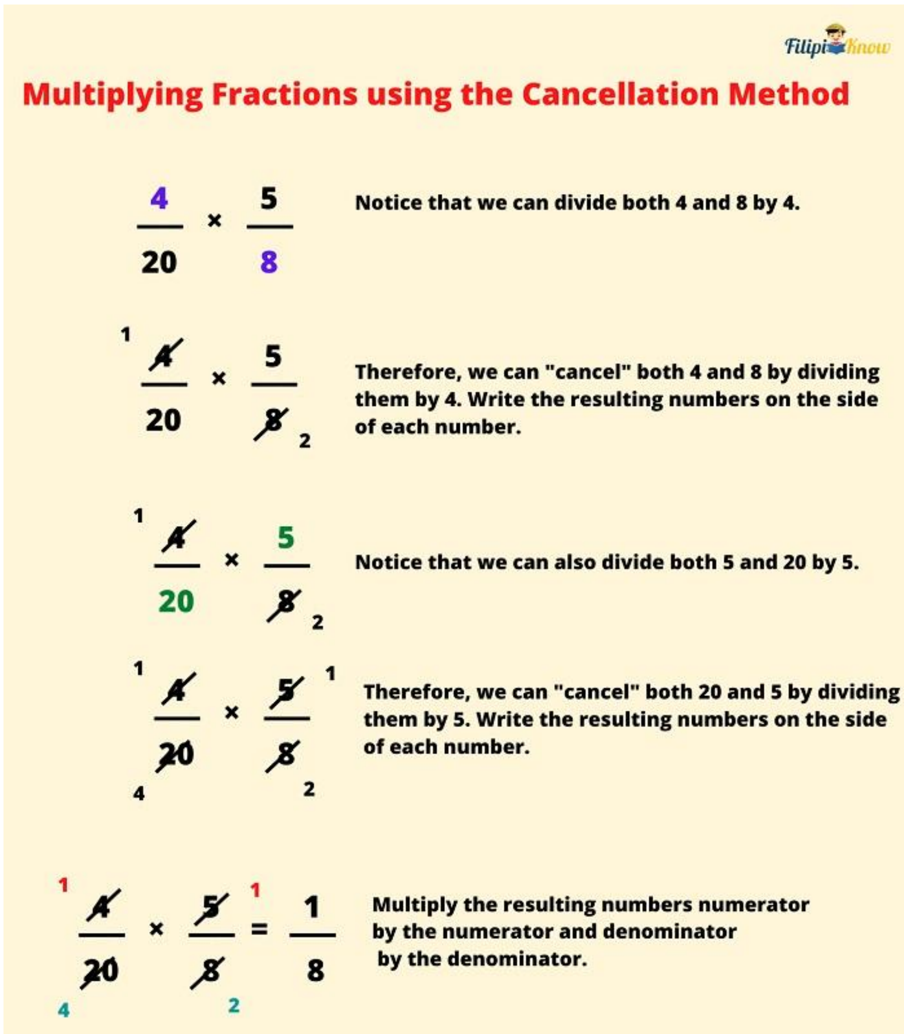
Hence, 3⁄4 of 100 is equal to 75.

Multiplying Fractions Through Cancellation Method.

We can actually make the process of multiplying fractions quicker through the cancellation method. In this method, we “cancel” numbers that have common factors so we can arrive at the product which is already in its lowest terms.

Example 1: Multiply 4⁄20 by 5⁄8.

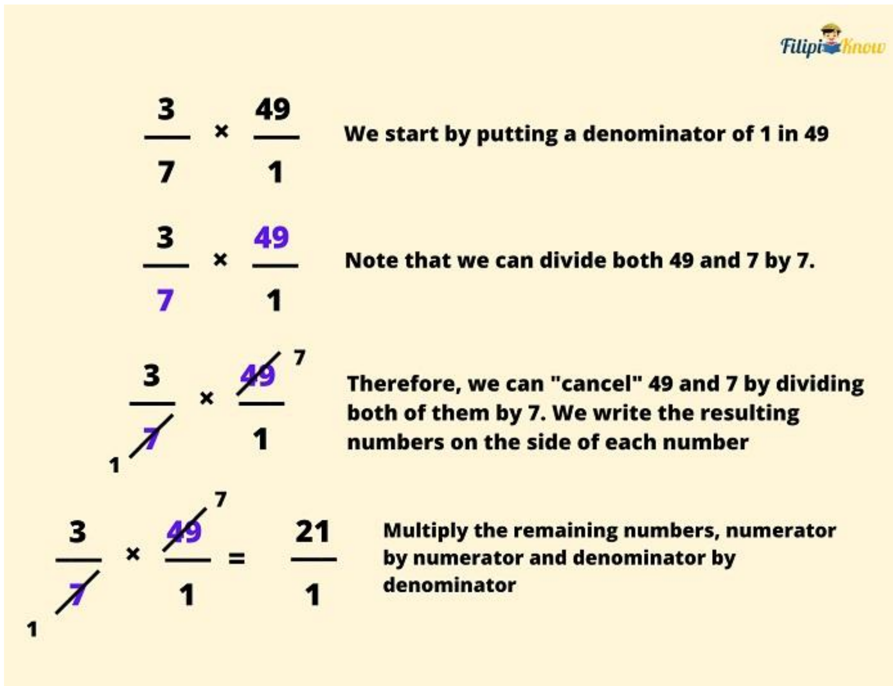
Solution:



Using the cancellation method, the answer is 1⁄8.

Example 2: What is 3⁄7 of 49?

Solution:



Again, the word “of” is a signal word for the multiplication of fractions.

Therefore, 3⁄7 of 49 is equal to 21.

Division of Fractions.

We are now on the fourth mathematical operation on fractions - division. However, before we proceed to the actual process of dividing fractions, let me introduce you first to the concept of the reciprocal or multiplicative inverse of a number.

Reciprocal or Multiplicative Inverse of a Number.

The reciprocal or multiplicative inverse of a fraction is the fraction that when multiplied by the original fraction, the result is 1. This definition sounds confusing and too technical so let me provide you with an easier way to grasp this concept.

Let's use fraction 5⁄6 as an example. The reciprocal of this fraction can be obtained by interchanging the positions of the numerator and the denominator. Therefore, the reciprocal of 5⁄6 is simply 6⁄5.

Easy, right? Now, can you determine the reciprocal of the following:

4⁄5, 5⁄8, and 25.

Here are the answers:

The reciprocal of 4⁄5 is 5⁄4.

The reciprocal of 5⁄8 is 8⁄5.

Meanwhile, the reciprocal of 25 is 25⁄1.

Let us go back to the definition of the reciprocal. The reciprocal or multiplicative inverse of a fraction is the fraction that when multiplied by the original fraction, the result is 1. This means that when you multiply a fraction by its reciprocal, the result is 1. For instance, when you multiply 4⁄5 by 5⁄4, you will obtain 1.

Now that you know what the reciprocal is, you are now prepared to proceed with the steps on how to divide fractions.

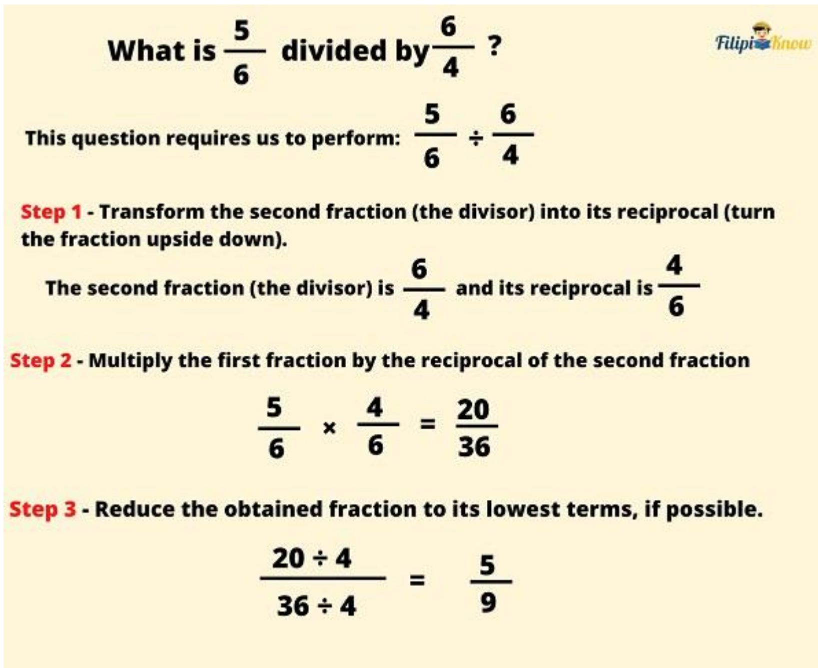
How to Divide Fractions.

Here are the steps you need to follow so you will be able to divide fractions:

1. Transform the second fraction (the divisor) into its reciprocal (turn the fraction upside down).
2. Multiply the first fraction by the reciprocal of the second fraction.
3. Reduce the obtained fraction to its lowest terms, if possible.

Example 1: What is 5⁄6 divided by 6⁄4?

Solution:



Therefore, $5/6 \div 6/4 = 5/9$

Example 2: Divide $3/7$ by $1/2$.

Solution:

Step 1: *Transform the second fraction (the divisor) into its reciprocal (turn the fraction upside down).*

The second fraction (the divisor) is $1/2$. Its reciprocal is $2/1$.

Step 2: *Multiply the first fraction by the reciprocal of the second fraction.*

$3/7 \times 2/1 = 6/7$

Step 3: *Reduce the obtained fraction to its lowest terms, if possible.*

$6/7$ is already in its lowest terms. Hence, we do not need to simplify it.

Therefore, 3⁄7 ÷ 1⁄2 = 6⁄7

Multiplication and Division of Mixed Numbers.

You already learned how to perform multiplication and division on fractions. This time, let us discuss how we can perform the same operations with mixed numbers.

When multiplying or dividing mixed numbers, the first thing you have to do is transform the given mixed numbers into improper fractions. Afterward, proceed with the steps on multiplying or dividing fractions.

Therefore, I suggest you review the steps on how to transform mixed numbers into improper fractions so you can multiply or divide mixed numbers with ease.

Let's have some examples:

Example 1: Multiply 1 2⁄3 by 2⁄5.

Solution:

The first thing you have to do is to transform the given mixed number into an improper fraction.

1 2⁄3 is a mixed number. If you transform it into an improper fraction, you have 5⁄3.

Afterward, you may now proceed with multiplying 5⁄3 by 2⁄5.

$$\frac{5}{3} \times \frac{2}{5} = \frac{10}{15}$$

Lastly, we can reduce 10⁄15 into its lowest terms:

$$\frac{10 \div 5}{15 \div 5} = \frac{2}{3}$$

Therefore, 1 2⁄3 × 2⁄5 = 2⁄3

Example 2: Divide 8 3⁄5 by 9.

Solution:

Start by transforming the given mixed number into an improper fraction.

8 3⁄5 = 43⁄5

Now, let's proceed to divide 43⁄5 by 9. The reciprocal of 9 is 1⁄9.

$$\frac{43}{5} \div 9 = \frac{43}{5} \times \frac{1}{9} = \frac{43}{45}$$

Therefore, 8 3⁄5 ÷ 9 = 43⁄45

Part II: Operations on Decimals.

If we can add, subtract, multiply, and divide fractions, we can also perform these operations with decimal numbers. In this section, let's discuss how to perform these mathematical operations with decimals.

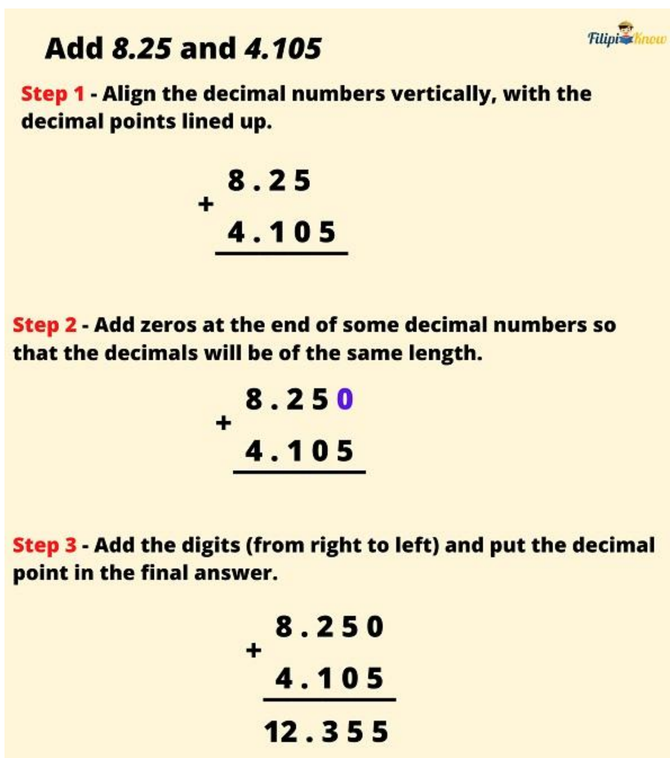
Addition and Subtraction of Decimals.

To add decimal numbers, follow these steps:

1. Align the decimal numbers vertically, with the decimal points lined up.
2. Add zeros at the end of some decimal numbers so that the decimals will be of the same length.
3. Add or subtract the digits and put the decimal point in the final answer.

Example 1: Delly bought a pencil worth PHP 8.25 and an eraser worth PHP 4.105. How much is the total amount of items that Delly bought?

Solution: We can answer this problem by adding the given amounts which are decimal numbers.



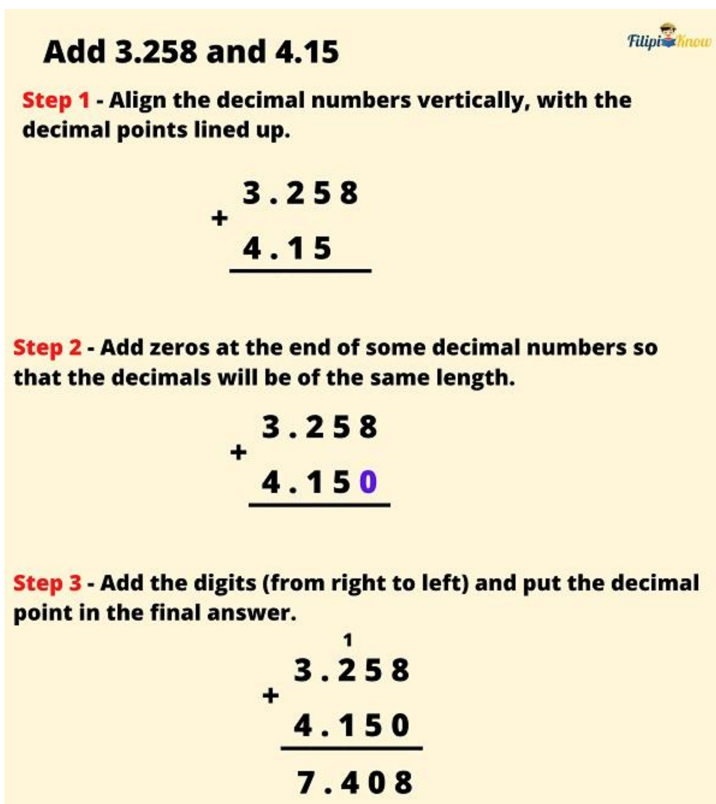
To solve this problem, we started by aligning the given decimal numbers. Afterward, we added a zero at the end of 8.25 so that it will be of the same length as 4.105. Lastly, we performed column addition from right to left (just like with whole numbers) and put the decimal point by bringing it down.

Therefore, 8.25 + 4.105 = 12.355

Example 2: Letty loves jogging. On Monday, she jogged a distance of 3.258 km. Meanwhile, on Tuesday, she jogged a distance of 4.15 km. What is the total distance covered by Letty on Monday and Tuesday?

Solution:

We can answer this problem by adding the given distances which are decimal numbers.

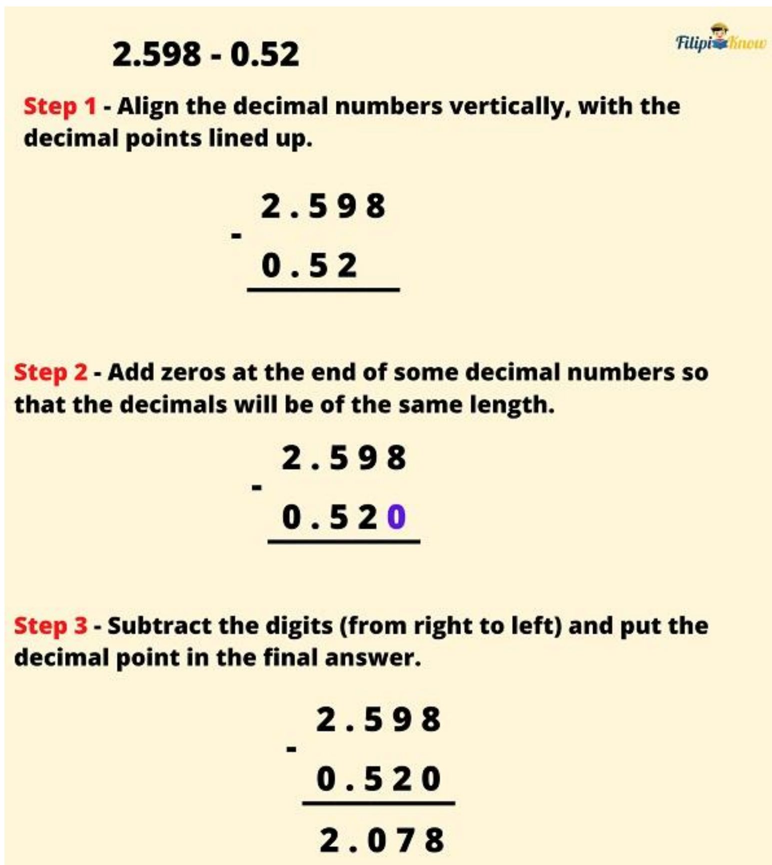


Therefore, Letty covered a total distance of 7. 408 km on Monday and Tuesday.

Example 3: Berto has 2.598 liters of alcohol. He used 0.52 liters for disinfecting his furniture. How many liters of alcohol were left?

Solution:

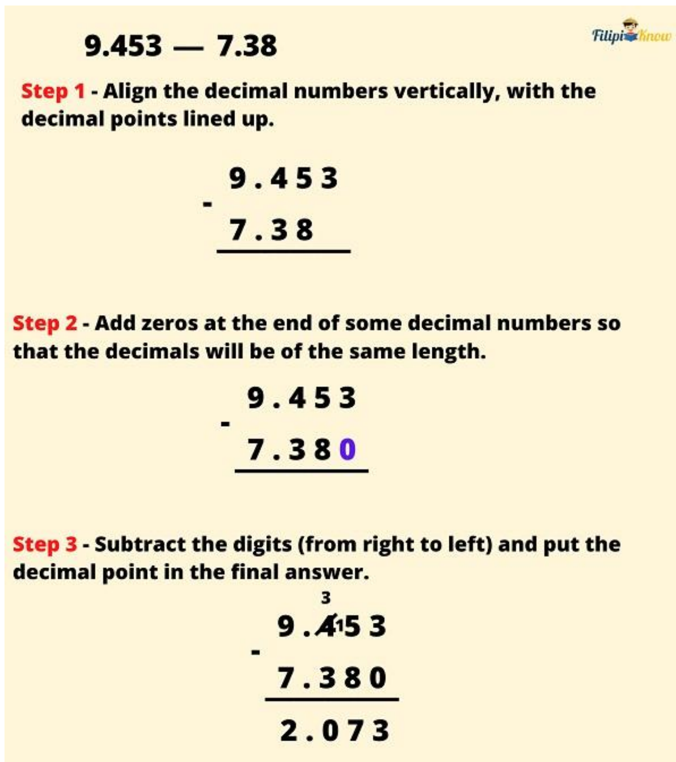
We can solve this problem by subtracting 0.52 from 2.598



Therefore, 2.078 liters of alcohol were left.

Example 4: What is the difference between 9.453 and 7.38?

Solution:



Thus, the difference between 9.453 and 7.38 is 2.073.

Multiplication of Decimals.

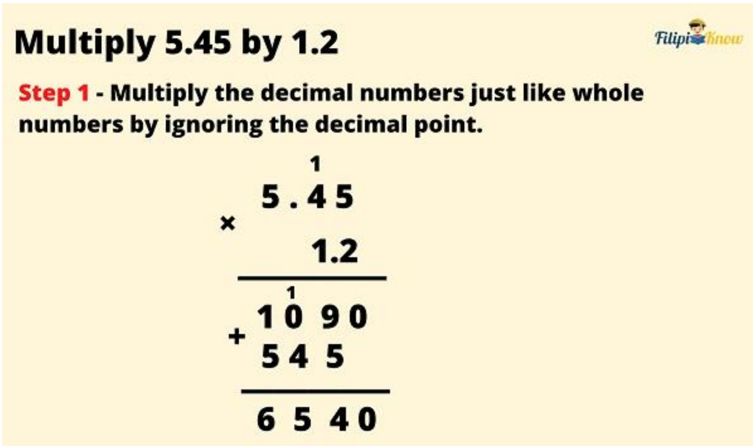
When multiplying decimal numbers, the first thing you have to do is to ignore the decimal point and multiply the digits just like whole numbers. Then, put the decimal point in the answer. The resulting number must have as many decimal places as the total number of decimal places the two original decimals have.

To understand the method mentioned above, let us have some examples:

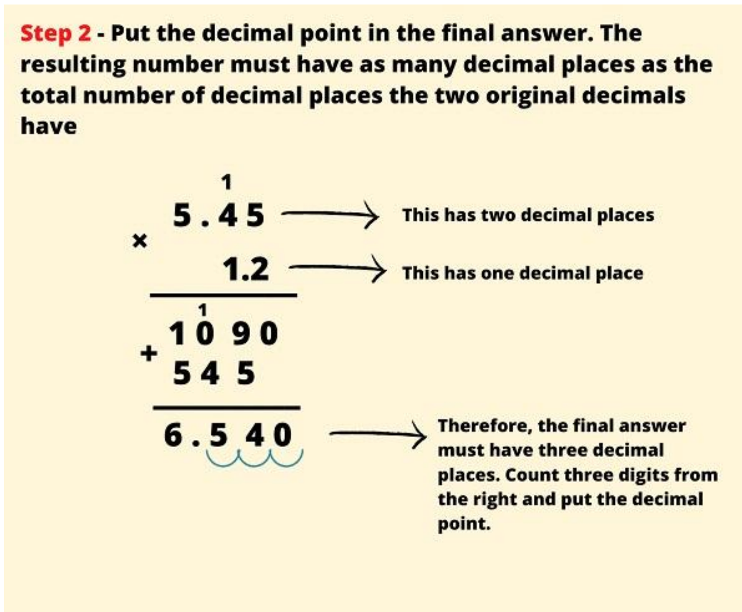
Example 1: Multiply 5.45 by 1.2

Solution:

We start our calculation by ignoring the decimal point and multiplying the numbers just like whole numbers.



We have obtained 6540 from Step 1 but it is not the final answer yet. We need to put the decimal point somewhere in its digits.



5.45 has two digits at the right of its decimal point. Thus, it has two decimal places. Meanwhile, 1.2 has one digit at the right of its decimal point. Therefore, it has one decimal place. The total number of decimal places we now have is three (two from 5.45 and one from 1.2). Thus, the final answer must have three decimal places.

To determine where we should put our decimal point in 6540, count three digits from the right then put the decimal point. Hence, the decimal point should be at 6.540

Thus, the answer is 6.540 or 6.54

Division of Decimals.

To divide decimal numbers, you may follow these steps:

1. Move the decimal point of the divisor (the second decimal) to the right until it becomes a whole number.

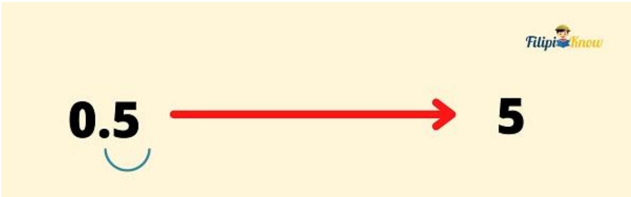
2. Move the decimal point in the dividend (the first decimal) to the right the same number of places you move the decimal point in the divisor.
3. Divide normally just like whole numbers using the new decimals obtained from Step 1 and 2 and put the decimal point to the final answer.

Let us apply these steps to our example below:

Example: Divide 32.95 by 0.5

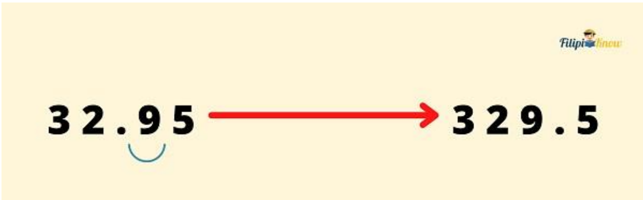
Solution:

Step 1: *Move the decimal point of the divisor (the second decimal) to the right until it becomes a whole number.*

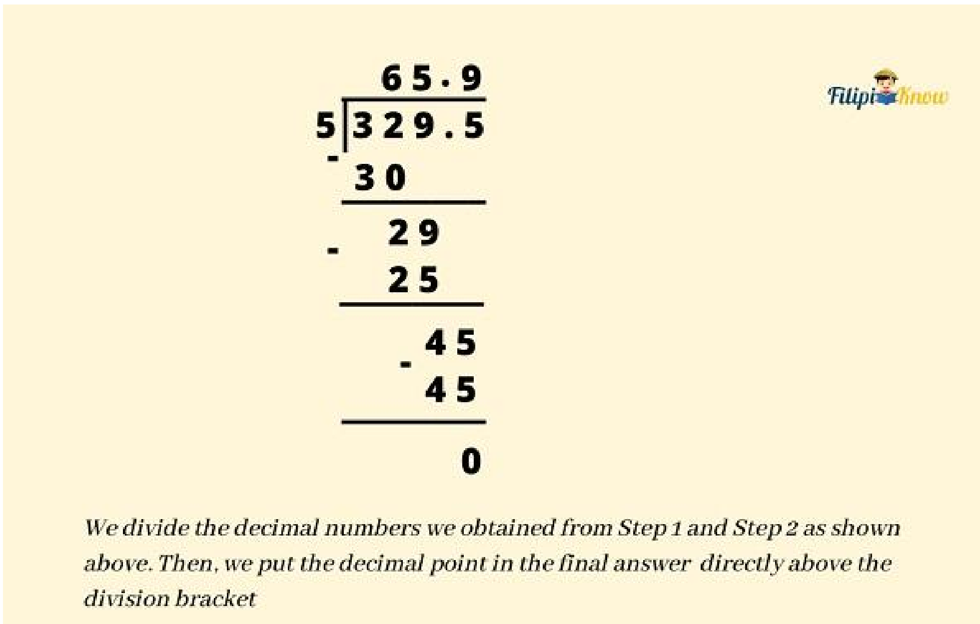


We can move one decimal place to the right of 0.5 so that it becomes a whole number (which is 5).

Step 2: *Move the decimal point in the dividend (the first decimal) to the right the same number of places you move the decimal point in the divisor.*



Step 3: *Divide normally just like whole numbers using the new decimals obtained from Step 1 and 2 and put the decimal point to the final answer.*



Therefore, 32.95 ÷ 0.5 = 65.9