We may have mastered the skill of comparing objects in a descriptive manner, but *how do we compare objects in a mathematical way?*

The answer is through ratio.

The ratio is the mathematical tool we use to compare quantities using division. The concept of ratios leads to the concept of proportions which has a lot of application in our daily lives such as when we convert currencies, estimate the volume of gasoline required for a car to cover a certain distance, calculate the cost of items bought, and so on.

In this reviewer, let us study what ratio and proportion are and how we can use them to solve real-life world problems.

Ratio.

A ratio shows how the quantity of an object is related to the quantity of another object. For example, if there are 15 male and 23 female students in a classroom, we can compare these quantities using a ratio, in particular, 15 : 23.

To express two quantities being compared as a ratio, we usually write it into the format below. Note that we use a colon (:) to express a ratio.

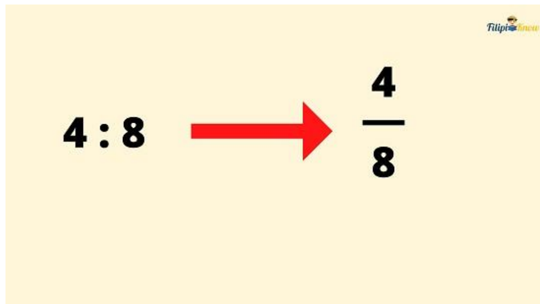
<first number> : <second number>

Example: *Aling Bela has 4 chickens and 8 pigs on her small farm. What is the ratio of her chickens to her pigs?*

Solution: We can express the ratio of chickens to pigs that Aling Bela owns as ***4 : 8***

Ratio as a Fraction.

We can write a ratio into its equivalent fractional form. We just write the first number as the numerator then write the second number as the denominator.



For instance, using our example above about Aling Nena's chickens and pigs, we can express the ratio of her chickens to her pigs *4 : 8* as 4⁄8.

Example 1: *For every 4 burgers you will buy, you have to pay PHP 128. What is the ratio of the number of burgers bought to the price you have to pay? Express the ratio in fractional form.*

Solution: We can express the ratio of the number of burgers bought to the price you have to pay as 4 : 128. In fractional form, we can write this as 4⁄128.

Example 2: *There are 15 science teachers in a public high school. In that same high school, there are 10 math teachers. What is the ratio of science teachers to math teachers in that public high school? Express the ratio in fractional form.*

Solution: We can express the ratio of science teachers to math teachers in that public high school as 15 : 10. In fractional form, we write it as 15⁄10.

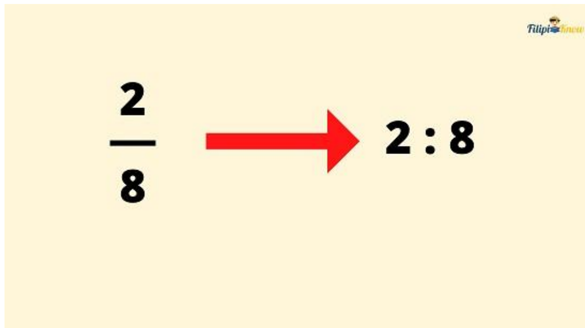
Using Ratio to Compare a Part to a Whole.

We have already defined what ratios are. However, the ratios that we have tackled in our previous sections pertain to a comparison of a quantity of an object to the quantity of a different object.

This time, let us use the ratio to compare a part of a whole to the whole itself.

Suppose that you and your friends bought a pizza and sliced it into 8 equal parts. Suppose that you're able to take 2 slices from it. *What is the ratio of the slices of pizza you have (a portion of the whole pizza) to the total number of slices (the whole pizza)?*

The given situation above might ring a bell to you. Yes, we can use fractions to show that comparison. In particular, fraction 2⁄8 can be expressed into a ratio as 2 : 8



This means that to use the ratio to compare a part of a whole to the whole itself, we can use this format:

<portion of the whole> : <total number portions of the whole>

Example: *In a classroom, 15 students are male while 20 students are female. What is the ratio of female students to the total number of students in the classroom?*

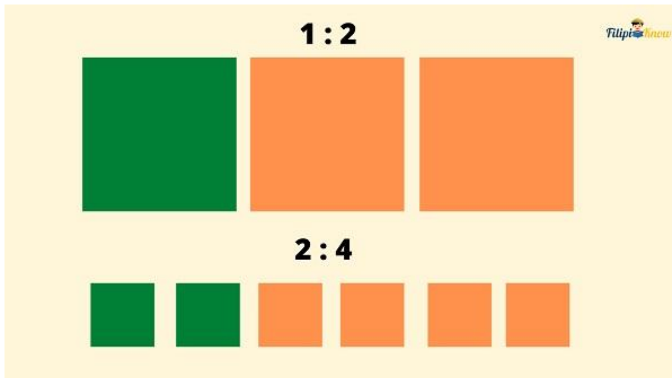
Solution: There are 20 female students in the classroom. Meanwhile, the total number of students in the classroom is the sum of the number of male students and the number of female students. In total, there are 15 + 20 = 35 students in that classroom.

Therefore, the ratio of female students to the total number of students in that classroom can be expressed as 20 : 35

Proportion.

A proportion indicates that the two ratios are equal. In other words, proportions are equivalent ratios. Hence, if we say that ratios are proportional, we mean that those ratios are equal in values.

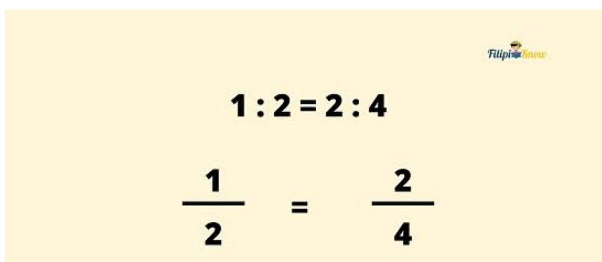
Suppose 1 : 2 and 2 : 4. We illustrate these ratios as shown below:



It's clearly seen that the ratios represent the same parts. It implies that these ratios are equivalent.

Hence, 1 : 2 = 2 : 4 is a proportion.

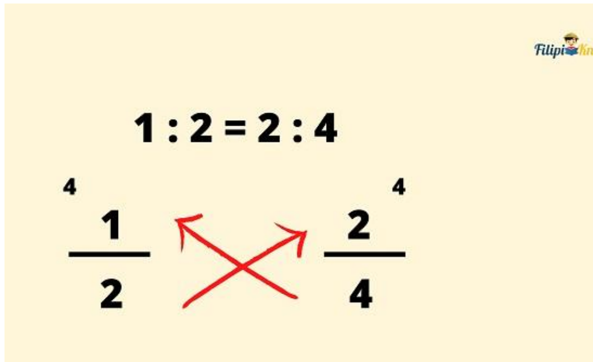
You might have realized that proportions are kind of similar to the concept of equivalent fractions. Indeed, proportions indicate equivalent fractions since we can write ratios in fractional form.



How to Know if Two Ratios are Proportional.

Two ratios are proportional if they are equal. One way to determine if two ratios are equal is by converting them into fractional form and then using the cross-multiplication method which we discussed in the **Fractions and Decimals** reviewer.

For instance, let us use the cross-multiplication method to determine if 1 : 2 = 2 : 4



Since the products are equal (both are equal to 4), then the ratios are equal. Hence, the ratios are proportional.

Moreover, you might also notice that if we multiply the numbers in a ratio, we can obtain another ratio that is proportional to that ratio.

For example, if we multiply each number in 5 : 2 by the same number, let's say 2, we have 10 : 4. Using the cross-multiplication method, you can verify that 5 : 2 = 10 : 4

Example: *Give a ratio that is equivalent or proportional to 2 : 9*

Solution: We can determine a ratio equivalent or proportional with 2 : 9 by multiplying each number in 2 : 9 by the same number.

Let us try to multiply the numbers in 2 : 9 by 5.

(2 x 5) : (9 x 5) = 10 : 45

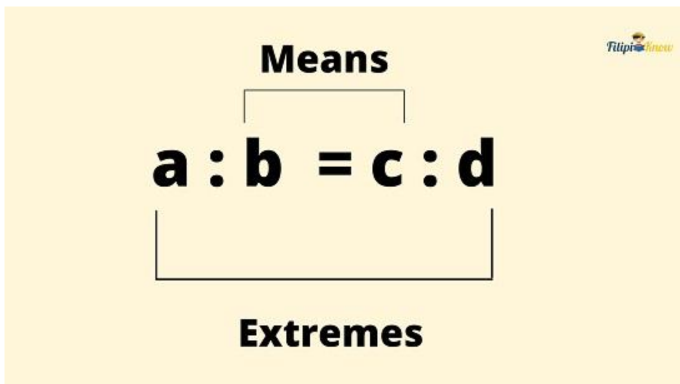
Hence, 2 : 9 = 10 : 45.

***Note**: The number that you can use to find a ratio that is proportional to 2 : 9 is arbitrary. If we multiply the numbers in 2 : 9 by the same number, we will come up with a ratio that is proportional to 2 : 9. In this example, I just arbitrarily used 5. You may use any number and multiply it to the numbers in 2 : 9 and you will come up with a ratio that is proportional to it. For example, I can multiply the numbers of 2 : 9 by 7 and obtain 14 : 63. 14 : 63 is also proportional with 2 : 9*

Parts of a Proportion: Extremes and Means.

Suppose a proportion *a : b = c : d* where *a, b, c,* and *d* represent [real numbers](https://filipiknow.net/real-numbers/).

The first and last terms (i.e., *a* and *d*) of the proportion are called the **extremes.** Meanwhile, the second and third terms (i.e., *b* and *c*) are called the **means**.



Example*: Determine the extremes and the means of the proportion 5 : 10 = 20 : 40*

Solution: The extremes are the first and last terms of the proportion which are 5 and 40, respectively. Meanwhile, the means are the second and third terms of the proportion which are 10 and 20, respectively.

Properties of Proportion.

Using the fact that proportions are equivalent ratios, we can mathematically derive its properties. These properties are very helpful when solving problems involving ratio and proportion.

Here are the properties of proportion:

1. The product of the means is equal to the product of the extremes.

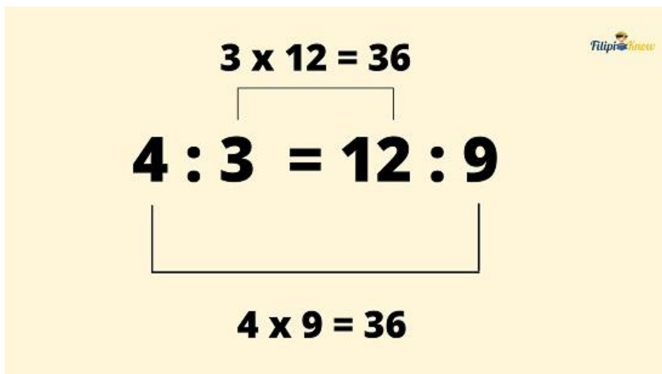
For every proportion a : b = c : d, then a x d = b x c

This property tells us that if we multiply the means and also multiply the extremes of a proportion, we will obtain the same number.

For example, suppose the proportion 4 : 3 = 12 : 9.

If we multiply the means: 3 x 12 = 36

If we multiply the extremes: 4 x 9 = 36



Note that the products of the means and the extremes are both equal to 36.

Example 1: *What must be N so that N : 8 = 2 : 16 is a proportion?*

Solution: Let us use the fact that the product of the means of a proportion is equal to the product of the extremes.

Multiplying the means, we have: 8 x 2 = 16

Multiplying the extremes, we have 16 x N

Now, by the first property, 16 x N = 16. What must be multiplied by 16 so that it will be 16? That number should be 1.

Hence, N = 1.

Therefore, the proportion should be 1 : 8 = 2 : 16.

Example 2: *Four kilos of chicken cost PHP 640. How many kilos of chicken can you buy with PHP 3 200?*

Solution: The ratio of the kilos of chicken that can be bought to the cost is 4 : 640. Now, let's use N to represent the number of kilos of chicken that can be bought with PHP 3200. Thus, we have the ratio N : 3200.

4 : 640 = N : 3200

Let us apply the fact that the product of the means is equal to the product of extremes so we can determine N.

Multiplying the means of the ratio: 640 x N

Multiplying the extremes of the ratio: 4 x 3200 = 12800

Since the product of the means is equal to the product of the extremes:

640 x N = 12800

What must be multiplied to 640 to obtain 12800? We determine that number by dividing 12800 by 640.

N = 12800 ÷ 640 = 20

Therefore, you can buy 20 kilos of chicken with PHP 3200.

2. The reciprocals of the ratios in a proportion are equal.

Recall that the reciprocal of a fraction is its multiplicative inverse, or simply the same fraction but with the positions of the numerator and the denominator reversed.

For example, the reciprocal of 2⁄5 is 5⁄2.

Given a proportion, say *a : b = c : d*, we can express it in fractional form as a⁄b = c⁄d

If we get the reciprocal of both fractions in a⁄b = c⁄d, we have:

b⁄a = d⁄c

We can express $b/a = d/c$ in ratio as *b : a = d : c*

This property states that if we take the reciprocal of each ratio in a proportion, the ratios are still proportional. In symbols:

$$a : b = c : d \rightarrow b : a = d : c$$

Example: *If 5 : 4 = 35 : 28, what should be N so that 4 : 5 = 28 : N*

Solution: Since the ratios in the proportion are reciprocated, we can use the second property of proportions. Using the second property, N = 35.

3. Switching the means or the extremes in a proportion will result in a proportion.

Suppose the proportion *1 : 7 = 3 : 21.* If we try to switch the positions of the means of this proportion, we have *1 : 3 = 7 : 21*. You can verify using cross-multiplication that *1 : 3 = 7 : 21* is true (that is, *1 : 3* and *7 : 21* are equivalent ratios or proportional*).*

Now, let us try switching the extremes of *1 : 7 = 3 : 21.* That is, we obtain *21 : 7 = 3 : 1*. Again, you can verify using cross-multiplication that *21 : 7 = 3 : 1* is true.

Hence, for every proportion *a : b = c : d*, switching the means or the extremes will still result in a proportion.

$$a : b = c : d \rightarrow a : c = b : d \text{ and } d : b = c : a$$

Example: *When A is divided by 5, the result will be equal to the result when you divide B by 2. What is the result if you divide A by B?*

Solution: The problem sounds tricky since we have no idea what the values of A and B are. However, using the third property of proportion, we can determine the result when we divide A by B.

A divided by 5 can be written as $A/5$, which can then be expressed into a ratio as *A : 5*.

Meanwhile, B divided by 2 can be written as $B/2$, which can then be expressed into a ratio as B : 2.

Since the problem states that if A is divided by 5, the result will be equal to the result if B is divided by 2, then

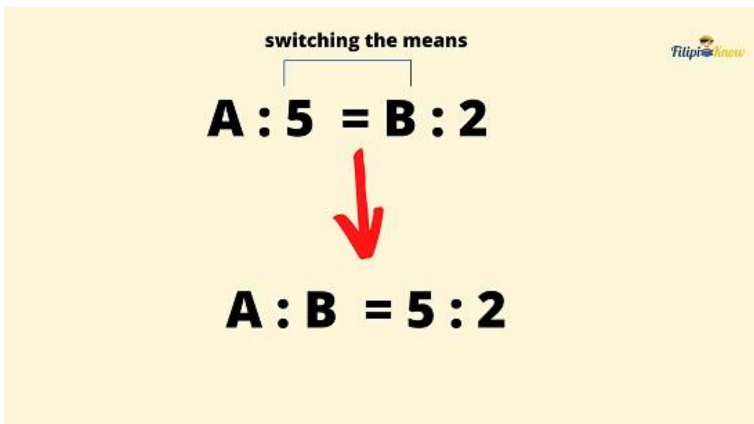
A : 5 = B : 2

We want to know, what will be the result when we divide A by B or A⁄B or, as a ratio, *A : B*

So, from *A : 5 = B : 2,* how can we obtain *A : B?*

We can apply the property that if we switch the means of a proportion, the result is still a proportion.

Let us now switch the means of *A : 5 = B : 2*



We obtain *A : B = 5 : 2.* Expressing into a fractional form:

A⁄B = 5⁄2

Therefore, if A is divided by B, the result is 5⁄2 or 2.5.

How to Solve Problems Involving Ratio and Proportion.

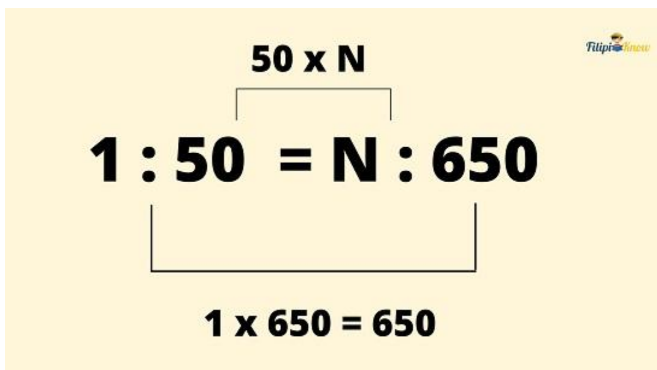
In this section, let us try to solve some real-life word problems that can be solved using the concepts of ratio and proportion.

Problem 1: *Suppose that on a certain date, 1 US dollar is equal to PHP 50. How many US dollars is equivalent to PHP 650?*

Solution: The ratio of US dollars to Philippine Peso can be expressed as 1 : 50. Let N be the number of dollars we can obtain from PHP 650.

Hence, we have this proportion: 1 : 50 = N : 650

The product of the means is equal to the product of extremes. Thus:



What must be multiplied to 50 to obtain 650?

N = 650 ÷ 50 = 13

Therefore, PHP 650 is equal to 13 US dollars.

Problem 2 : *Leonor loves animals. In fact, he has a lot of dogs and cats in his house. The ratio of dogs to his cats is 1 : 3. The total number of dogs and cats is 8. How many cats does Leonor own?*

Solution: The ratio of dogs to cats is 1 : 3. This doesn't mean that Leonor has 1 dog and 3 cats. 1 : 3 is just a ratio used to compare the quantity of dogs to cats. To find the actual number of dogs and cats that Leonor has, we need to find two numbers with a sum of 8 that when expressed as a ratio, will be proportional to 1 : 3.

Recall that we can obtain a ratio that is proportional to 1 : 3 if we multiply both 1 and 3 by the same number.

Let us multiply the parts of the ratio with a number *a.*

(1 x a) : (3 x a)

This means that we have two numbers *1 x a* and *3 x a*. *1x a* represents the total number of dogs that Leonor has while *3 x a* represents the total number of cats that Leonor has.

Since the total number of dogs and cats that Leonor has is 8:

(1 x a) + (3 x a) = 8

We can simplify the expression above as:

(4 x a) = 8

What must be multiplied by 4 to obtain 8? Simple, that number must be 2.

Hence, a = 2.

Recall that 1 x a represents the number of dogs that Leonor has. Since we have computed that a = 2, then Leonor has 1 x (2) = 2 dogs.

Doing the same thing to find the number of cats that Leonor has: 3 x (2) = 6 cats.

Therefore, Leonor has 6 cats.