In the previous chapter, we reviewed the **linear equations** and how we can use them to solve some word problems.

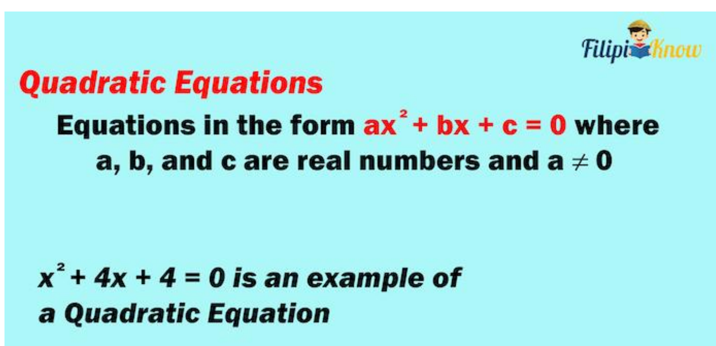
However, there are word problems where linear equations are not enough to describe the given situation. In cases like these, we need another type of equation: the quadratic equations.

In this reviewer, we'll explore the definition of a quadratic equation, the different ways to solve it, and how we can apply it to solve word problems.

What Are Quadratic Equations?

Quadratic equations are equations in the form $ax^2 + bx + c = 0$ where *a, b,* and *c* are real numbers and *a* is not equal to 0.

In simple words, a quadratic equation has two as the highest exponent of its variable. For example, $x^2 + 4x + 4 = 0$ is a quadratic equation since the highest exponent of the variable in this equation is 2.



Example: *Which of the following are quadratic equations?*

a) $x^2 - 2x + 1 = 0$

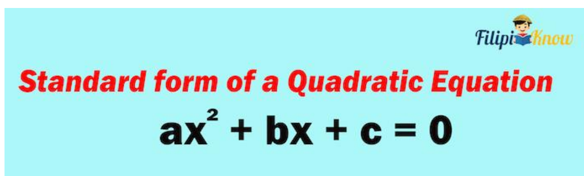
b) $x^2 = 9$

c) x + 2 = -2

Solution: The equations in letters *a* and *b* are quadratic equations since the highest exponent of their *x* (or variable) is 2. On the other hand, *c* is not a quadratic equation since the highest exponent of its *x* (or variable) is 1, making it a linear equation.

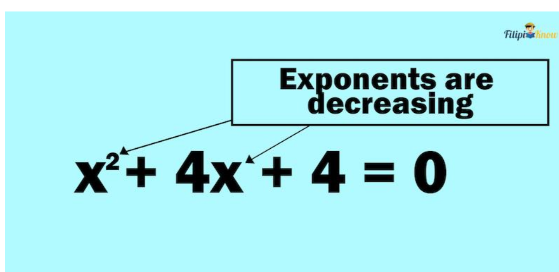
Standard Form of a Quadratic Equation.

The standard form of a quadratic equation is $ax^2 + bx + c = 0$



When we say that a quadratic equation is in standard form, it means that the terms of the equation are arranged in a manner where the exponents of the variable are decreasing.

For example, $x^2 + 4x + 4 = 0$ is in standard form because the terms are arranged in a manner where the exponents of the variable are in decreasing order.



But why is it important to know if a quadratic equation is in standard form?

The answer is because **if a quadratic equation is in standard form $ax^2 + bx + c = 0$, we can easily determine the values for a, b, and c.**

a, b, and ***c*** are the real number parts of the equation.

$$x^2 + 4x + 4 = 0$$

$$a = 1, b = 4, c = 4$$

Take a look again at this equation: $x^2 + 4x + 4 = 0$. We already know that this quadratic equation is in standard form.

The *a* of a quadratic equation in standard form is the numerical coefficient of the quadratic term or the term with x^2. In $x^2 + 4x + 4 = 0$, the quadratic term is x^2 and its numerical coefficient is 1. Thus $a = 1$

The *b* of a quadratic equation in standard form is the numerical coefficient of the linear term or the term with *x*. In $x^2 + 4x + 4 = 0$, the linear term is $4x$ and its numerical coefficient is 4. Thus, $b = 4$.

Lastly, **the *c* of a quadratic equation in standard form is the constant term or the term without the *x*.** In $x^2 + 4x + 4 = 0$, the constant term is 4. Thus, $c = 4$.

Therefore, in $x^2 + 4x + 4 = 0$, the values of *a, b,* and *c* are: $a = 1$, $b = 4$, and $c = 4$.

Example: *Determine the values of a, b, and c (the real number parts) in $2x^2 + 4x - 1 = 0$*

Solution: Since the $2x^2 + 4x - 1 = 0$ is already in standard form, then the values of *a, b*, and *c* are easy to determine:

- a = 2 (the numerical coefficient of $2x^2$)
- b = 4 (the numerical coefficient of 4x)

- c = -1 (the constant term is -1)

The *a, b,* and *c* of a quadratic equation can be determined only once we have expressed it in standard form $ax^2 + bx + c = 0$. If a quadratic equation is not yet in the standard form, we cannot immediately tell the values of *a, b,* and *c*.

Later in this reviewer, you'll learn the importance of determining the values of *a, b,* and *c* of a quadratic equation especially when you start solving them using the quadratic formula.

Different Forms of Quadratic Equation.

Not every quadratic equation that you will encounter and solve is in standard form. Quadratic equations appear in different forms. It is important to learn about them since there are specific techniques that we can use to solve equations in these forms.

$ax^2 = c$ or $ax^2 + c = 0$ Form.

Quadratic equations such as $x^2 = 9$, $2x^2 = 16$, $5x^2 = 10$, $-x^2 = -1$ are in the form $ax^2 = c$. As you might have noticed, quadratic equations in this form do not have a linear term or a term with a variable raised to 1.

Quadratic equations in the form $ax^2 = c$ can also appear in the form of $ax^2 + c = 0$.

For example, we know that $x^2 = 9$ is a quadratic equation in $ax^2 = c$ form. However, if we transpose 9 to the left-hand side of the equation:

$x^2 = 9$

$x^2 - 9 = 0$ *Transposition Method*

This shows that $x^2 = 9$ is the same as $x^2 - 9 = 0$. In other words, quadratic equations in $ax^2 = c$ form can also appear in the form $ax^2 + c = 0$.

The technique we usually use to solve quadratic equations in these forms is by extracting the square root. You will learn more about this method in the succeeding sections.

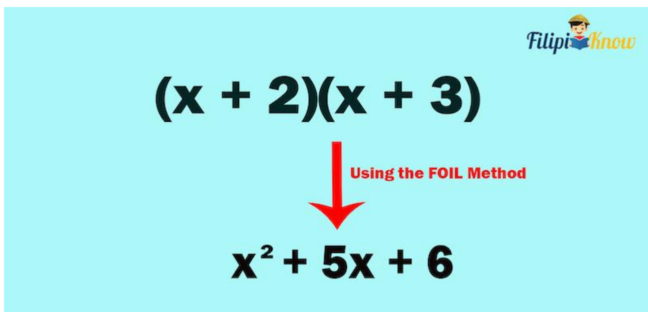
(x + a)(x + b) Form or the Factored Form.

In some quadratic equations, the exponent of 2 in the variable can't be identified unless you perform some computations first.

For instance, *(x + 2)(x + 3) = 0* can be considered as a quadratic equation.

You're probably asking: *"But there's no* x^2 *in (x + 2)(x + 3) = 0, so why is it a quadratic equation?"*

Try to perform the **FOIL method** on *(x + 2)(x + 3)* and let's see what we'll obtain:



(x + 2)(x + 3) when multiplied is equal to $x^2 + 5x + 6$. Thus, the equation *(x + 2)(x + 3) = 0* is actually $x^2 + 5x + 6 = 0$. This is the reason why *(x + 2)(x + 3) = 0* is a quadratic equation.

Here are more examples of a quadratic equation in *(x + a)(x + b)* form or factored form:

- (x + 1)(x - 3) = 0
- (x - 2)(x + 1) = 0
- (x - 1)(x - 1) = 0

Example: *Which of the following are quadratic equations?*

a) (x + 2)(x - 1) = 0

b) $x^2 + x^3 = -9$

c) $2x^2 + 3x = -1$

d) $x^2 = 1$

Solution:

- Equation *a* is a quadratic equation in factored form.
- Equation *b* is NOT a quadratic equation since the highest exponent of its variable is 3.
- Equation *c* is a quadratic equation but not yet in standard form. We can transpose -1 to the left side so that it will be in standard form.
- Equation *d* is a quadratic equation in $ax^2 = c$ form.

Thus, equations *a*, *c*, and *d* are all quadratic equations.

How to Solve Quadratic Equations.

Let us discuss in this section the different methods of solving quadratic equations.

Method 1: How to Solve Quadratic Equation by Extracting Square Roots.

We usually use this method to solve for *x* of quadratic equations that are in the $ax^2 = c$ or $ax^2 + c = 0$ form.

Here are the steps to solve quadratic equations by extracting the square root:

1. Isolate the square variable (x^2) from other quantities. This means that x^2 must be the only quantity on the left-hand side and other quantities must be on the right-hand side.
2. Take the square root of both sides of the equation.

Example 1: *Solve for x in* $x^2 = 9$

Solution:

Step 1: Isolate the square variable (x^2) from other quantities.

x^2 is the only quantity on the left-hand side of $x^2 = 9$. This means that x^2 is already isolated from other quantities. Thus, we can skip this step.

Step 2: Take the square root of both sides of the equation.

We get the square root of both sides of the equation.

$$x^2 = 9$$
$$\sqrt{x^2} = \sqrt{9}$$
$$x = \pm 3$$
$$x_1 = 3 \quad x_2 = -3$$

Notes:

- If we get the square root of x^2, the result will be x.
- There are two square roots of a number: a positive root and a negative root, the reason why we put the sign ± when we take the square root of a number.

Thus, the answers are **$x_1 = 3$ and $x_2 = -3$**

The solutions of a quadratic equation are also called the **roots of a quadratic equation**. Thus, when we say the roots of $x^2 = 9$, we are referring to the solution of $x^2 = 9$.

Example 2: *What are the roots of $2x^2 = 8$?*

Solution:

Step 1: Isolate the square variable (x^2) from other quantities.

To remove the numerical coefficient and make x^2 the only quantity in the left-hand side of the equation, we can divide both sides of the equation by 2.

$2x^2/2 = 8/2$

$x^2 = 4$

Step 2: Take the square root of both sides of the equation.

$x^2 = 4$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

Thus, the roots of the equation are $\mathbf{x_1 = 2}$ **and** $\mathbf{x_2 = -2}$

Example 3: *What are the roots of* $x^2 + 4 = 20$?

Solution:

Although the given equation seems to be not in the $ax^2 = 0$ or $ax^2 + c = 0$ form, we can manipulate the equation so that we can solve it by extracting the square root.

Step 1: Isolate the square variable (x^2**) from other quantities.**

To isolate x^2 from other quantities, we can transpose 4 to the right-hand side of the equation:

$x^2 + 4 = 20$

$x^2 = -4 + 20$

$x^2 = 16$

Step 2: Take the square root of both sides of the equation.

$x^2 = 16$

$\sqrt{x^2} = \sqrt{16}$

$x = \pm 4$

Thus, the roots of the equation are $\mathbf{x_1 = 4}$ **and** $\mathbf{x_2 = -4}$

Example 4: *Solve for the roots of* $2x^2 - 6 = 0$

Solution:

Step 1: Isolate the square variable (*x^2*) from other quantities.

To isolate x^2 from other quantities, we can transpose -6 to the right-hand side of the equation:

$2x^2 - 6 = 0$

$2x^2 = 6$

$2x^2/2 = 6/2$

$x^2 = 3$

Step 2: Take the square root of both sides of the equation.

$x^2 = 3$

$\sqrt{x^2} = \sqrt{3}$

$x = \pm \sqrt{3}$

$\sqrt{3}$ is not a perfect square number (there's no integer multiplied to itself will give 3). Thus, we just write it as $\sqrt{3}$.

Thus, the roots of the equation are **$x_1 = \sqrt{3}$ and $x_2 = -\sqrt{3}$**

Example 5: *Solve for x in* $x^2 - 19 = 6$

Solution:

$x^2 - 19 = 6$

$x^2 = 19 + 6$ *Transposition Method*

$x^2 = 25$

$\sqrt{x^2} = \sqrt{25}$ *Taking the square root of both sides*

$x = \pm 5$

The values of *x* are **5 and -5**

Example 6: *Solve for x in* $(x + 4)^2 = 9$

Solution:

Note that it is much easier if we start extracting the square root of both sides first so that the resulting equation is just a linear equation:

$(x + 4)^2 = 9$

$\sqrt{(x + 4)^2} = \sqrt{9}$ *Taking the square root of both sides*

$x + 4 = \pm 3$

Since we have two square roots for 9, we are going to have to solve two linear equations:

- **Equation 1**: $x + 4 = 3$
- **Equation 2**: $x + 4 = -3$

Solving for Equation 1 first:

$x + 4 = 3$

$x = - 4 + 3$ *Transposition Method*

$x = - 1$

Thus, the first root is -1

Solving for Equation 2:

$x + 4 = - 3$

x = - 4 + (-3)

x = - 7

Thus, the second root is -7

Therefore, the roots of the quadratic equation are **- 1 and - 7**.

Method 2: How to Solve Quadratic Equation by Factoring.

In this method, we are going to apply what you have learned about factoring, specifically the method of factoring quadratic trinomials.

To solve quadratic equations by factoring, follow these steps:

1. Express the given equation in standard form. This means that one side of the quadratic equation must be 0.
2. Factor the expression. You must come up with two factors after factoring.
3. Equate each factor to zero and solve each resulting equation.

Example 1: *Solve for the roots of* $x^2 + 5x + 4 = 0$ *by factoring.*

Solution:

Step 1: Express the given equation in standard form. The given equation is already in standard form since it is in $ax^2 + bx + c = 0$ form and one of the sides of the quadratic equation is already 0. Hence, we can skip this step.

Step 2: Factor the expression. To factor $x^2 + 5x + 4$, follow the steps on factoring quadratic trinomials.

$x^2 + 5x + 4 = 0$

$(x + 4)(x + 1) = 0$ *by Factoring*

As shown above, we can come up with two factors which are $x + 4$ and $x + 1$.

Step 3: Equate each factor to zero and solve each resulting equation. We have obtained *x + 4* and *x + 1* as factors of $x^2 + 5x + 4$. We set both of these factors to zero. This means that we are going to have two linear equations.

- **Equation 1**: x + 4 = 0
- **Equation 2**: x + 1 = 0

Solving the equations above

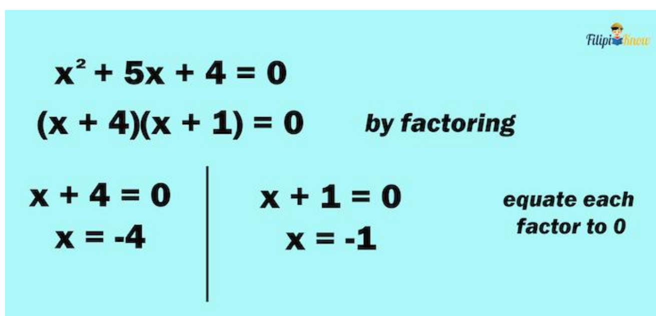
Equation 1: Equation 2:

x + 4 = 0 x + 1 = 0

x = - 4 x = - 1

Thus, the roots of the equation are $\mathbf{x_1 = -4}$ **and** $\mathbf{x_2 = -1}$.

Here's a quick preview of what we have done above:



Example 2: *Solve for the values of x in* $x^2 - 7x = -10$

Solution:

Step 1: Express the given equation in standard form. To express $x^2 - 7x = -10$, we have to transpose - 10 to the left-hand side so that the right-hand side will be 0:

$x^2 - 7x = -10$

$x^2 - 7x + 10 = 0$ *Transposition Method*

Step 2: Factor the expression. Factoring $x^2 - 7x + 10$ will give us $(x - 5)(x - 2)$

$x^2 - 7x + 10 = 0$

$(x - 5)(x - 2) = 0$ *by factoring*

Step 3: Equate each factor to zero and solve each resulting equation. We have obtained $x - 5$ and $x - 2$ as the factors of $x^2 - 7x + 10$. We set both of these factors to zero. This means that we are going to have two linear equations.

- **Equation 1:** $x - 5 = 0$
- **Equation 2:** $x - 2 = 0$

Solving the equations above

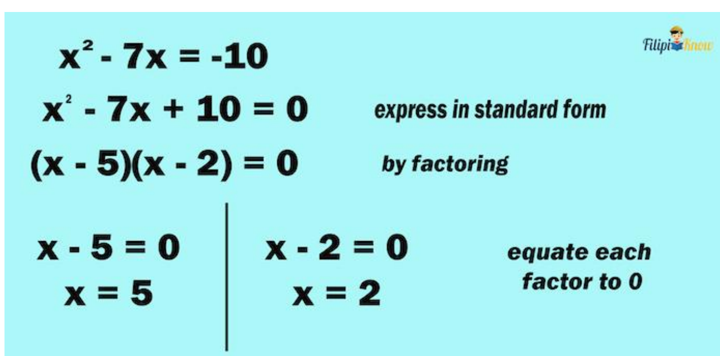
Equation 1: Equation 2:

$x - 5 = 0$ $x - 2 = 0$

$x = 5$ $x = 2$

Thus, the roots of the equation are $\mathbf{x_1 = 5}$ **and** $\mathbf{x_2 = 2}$.

Here's a quick review of our solution above:



Example 3: *Solve for the roots of* $2x^2 + 3x - 2 = 0$ *by factoring.*

Solution:

Step 1: Express the given equation in standard form. Since $2x^2 + 3x - 2 = 0$ is already in $ax^2 + bx + c = 0$ form and one of its side is already 0, we can skip this step.

Step 2: Factor the expression. You must come up with two factors after factoring. Factoring $2x^2 + 3x - 2$ will give us $(2x - 1)(x + 2)$:

$2x^2 + 3x - 2 = 0$

$(2x - 1)(x + 2) = 0$ *by factoring*

Step 3: Equate each factor to zero and solve each resulting equation. We have obtained $2x - 1$ and $x + 2$ as the factors of $2x^2 + 3x - 2$. We set both of these factors to zero. This means that we are going to have two linear equations.

- **Equation 1:** $2x - 1 = 0$
- **Equation 2:** $x + 2 = 0$

Solving the equations above

Equation 1: Equation 2:

$2x - 1 = 0$ $x + 2 = 0$

$2x = 1$ $x = -2$

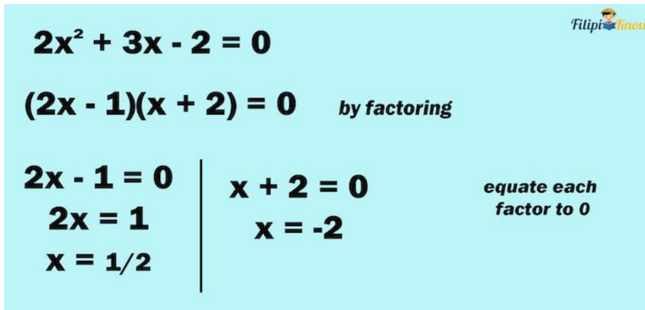
To cancel the numerical coefficient in Equation 1, we divide both of its sides by 2:

$2x/2 = \frac{1}{2}$

$x = \frac{1}{2}$

Thus, the roots of the equation are $\mathbf{x_1 = \frac{1}{2}}$ **and** $\mathbf{x_2 = -2}$.

Here's a quick review of what we have done above:



Factoring seems to be a powerful technique to solve quadratic equations. However, not every quadratic equation can be factored. For instance, $x^2 + x + 2 = 0$ cannot be solved by factoring since there are no two integers whose product is 2 and have a sum of 1.

But don't worry, there are other methods available that we can use in case a quadratic equation is not factorable. These methods are *completing the square method* and *quadratic formula*.

Method 3: How to Solve Quadratic Equations by Completing the Square.

This method is advisable to use if a quadratic equation is non-factorable.

To solve quadratic equations by completing the square, follow these steps:

1. Put the terms with variable x on the left-hand side of the equation while the constant term on the right-hand side.
2. Divide both sides of the equation by a (or the coefficient of the quadratic term).
3. Divide the b (or the coefficient of the linear term) by 2 and square the result. Add the result to both sides of the equation.
4. Factor the left-hand side of the equation. Express the factors as a square of a binomial.
5. Take the square root of both sides of the equation.
6. Solve the resulting linear equations.

Don't get intimidated by the steps above because we are going to discuss each one in detail in our examples below.

Example 1: *Solve for the roots of* $x^2 + 8x - 10 = 0$

Solution:

A closer look will reveal that $x^2 + 8x - 10$ is non-factorable since there are no two integers whose product is - 10 and have a sum of 8. For this reason, we are going to solve $x^2 + 8x - 10 = 0$ by completing the square.

Step 1: Put the terms with variable *x* on the left-hand side of the equation while the constant term on the right-hand side. In this case, transpose - 10 (i.e., the constant) to the right-hand side of the equation while all terms with *x* stay on the left-hand side.

$x^2 + 8x - 10 = 0$

$x^2 + 8x = 10$ *Transposition Method*

Step 2: Divide both sides of the equation by *a* (or the coefficient of the quadratic term). The *a* in $x^2 + 8x = 10$ is the coefficient of x^2. The coefficient of x^2 is 1. If we divide $x^2 + 8x = 10$ by 1, the result will still be $x^2 + 8x = 10$.

Thus, we can skip this step.

Step 3: Divide the *b* (or the coefficient of the linear term) by 2 and square the result. Add the result to both sides of the equation. The *b* of $x^2 + 8x = 10$ is the coefficient of $8x$ which is 8. Thus, $b = 8$.

Divide *8* by 2 ($8 \div 2 = 4$)

Square the result ($4^2 = 16$)

The number we obtain is 16.

We add 16 to both sides of $x^2 + 8x = 10$:

$x^2 + 8x = 10$

$x^2 + 8x + 16 = 10 + 16$ *Adding 16 to both sides of the equation*.

$x^2 + 8x + 16 = 26$

We have obtained the equation $x^2 + 8x + 16 = 26$. After you apply this step, you will obtain a perfect square trinomial (i.e., $x^2 + 8x + 16$). As we have learned from a previous chapter, perfect square trinomials can be factored.

Step 4: Factor the left-hand side of the equation. Express the factors as a square of a binomial. The left-hand side of $x^2 + 8x + 16 = 26$ is $x^2 + 8x + 16$. This can be factored as $(x + 4)(x + 4)$. We express $(x + 4)(x + 4)$ as $(x + 4)^2$

To summarize:

$x^2 + 8x + 16 = 26$

$(x + 4)(x + 4) = 26$ *by factoring*

$(x + 4)^2 = 26$ *Expressing as a square of a binomial*

Step 5: Take the square root of both sides of the equation.

Let us take the square root of both sides of the equation we have obtained from the previous step:

$(x + 4)^2 = 26$

$\sqrt{(x + 4)^2} = \sqrt{26}$ *Taking the square root of both sides*

$x + 4 = \pm\sqrt{26}$

Since 26 have two square roots (i.e., a positive and a negative square root), we have two linear equations:

- **Equation 1:** $x + 4 = \sqrt{26}$
- **Equation 2:** $x + 4 = -\sqrt{26}$

Step 6: Solve the resulting linear equations.

Solving each linear equation:

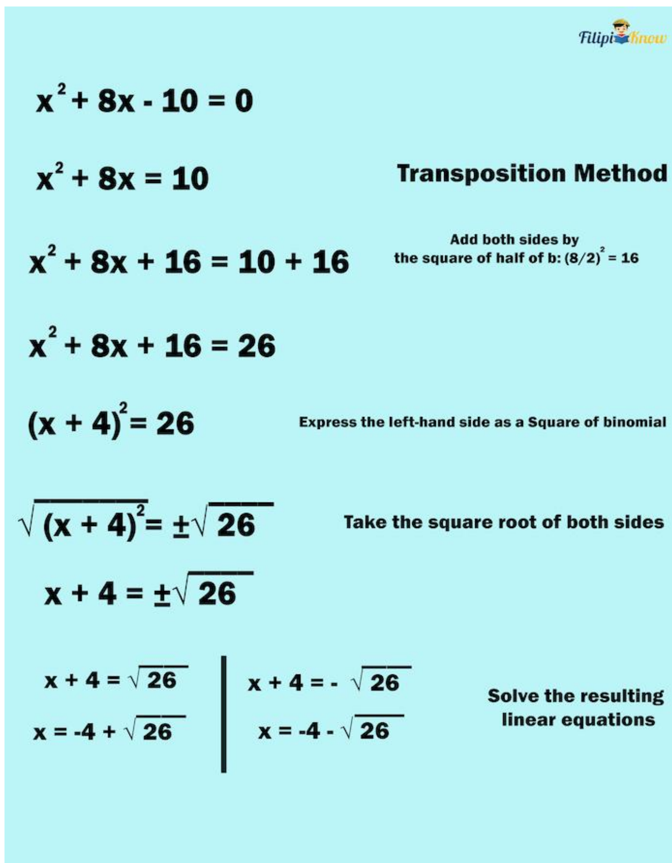
Equation 1: Equation 2:

$x + 4 = \sqrt{26}$ $x + 4 = -\sqrt{26}$

$x = -4 + \sqrt{26}$ $x = -4 - \sqrt{26}$

Therefore, the roots of the equation $x^2 + 8x - 10 = 0$ are $\mathbf{x_1 = -4 + \sqrt{26}}$ **and** $\mathbf{x_2 = -4 - \sqrt{26}}$

Here's a quick review of what we have done above:



Example 2: *Solve for the roots of* $2x^2 + 12x + 14 = 0$

Solution:

Since $2x^2 + 12x + 14 = 0$ is not factorable, let us solve this quadratic equation by completing the square.

Step 1: Put the terms with variable *x* on the left-hand side of the equation while the constant term on the right-hand side. By transposing 14 (i.e., the constant) to the right-hand side of the equation, all terms with x will remain on the left-hand side.

$2x^2 + 12x + 14 = 0$

$2x^2 + 12x = - 14$ *Transposition Method*

Step 2: Divide both sides of the equation by *a* (or the coefficient of the quadratic term). The a in $2x^2 + 12x + 14 = 0$ is the coefficient of x^2. The coefficient of $2x^2$ is 2. Thus,

$2x^2 + 12x = - 14$

$[2x^2 + 12x] \div 2 = -14 \div 2$ *Dividing both sides of the equation by a = 2*

$x^2 + 6x = - 7$

Step 3: Divide the *b* (or the coefficient of the linear term) by 2 and square the result. Add the result to both sides of the equation. The b of $x^2 + 6x = - 7$ is the coefficient of $6x$ which is 6. Thus, $b = 6$.

Divide 6 by 2 ($6 \div 2 = 3$)

Square the result ($3^2 = 9$)

The number we obtain is 9.

We add 9 to both sides of $x^2 + 6x = -7$:

$x^2 + 6x = - 7$

$x^2 + 6x + 9 = - 7 + 9$ *Adding 9 to both sides of the equation*

$x^2 + 6x + 9 = 2$

We have obtained the equation $x^2 + 6x + 9 = 2$. After completing this step, you will obtain a perfect square trinomial. $x^2 + 6x + 9$ is a perfect square trinomial. As we learned from a previous chapter, perfect square trinomials can be factored.

Step 4: Factor the left-hand side of the equation. Express the factors as a square of a binomial. The left-hand side of the equation is $x^2 + 6x + 9$. This can be factored as $(x + 3)(x + 3)$. We express $(x + 3)(x + 3)$ as $(x + 3)^2$

$x^2 + 6x + 9 = 2$

$(x + 3)(x + 3) = 2$ *by factoring*

$(x + 3)^2 = 2$ *Expressing as a square of a binomial*

Step 5: Take the square root of both sides of the equation.

Let us take the square root of both sides of the equation we have obtained from the previous step:

$(x + 3)^2 = 2$

$\sqrt{(x + 3)^2} = \sqrt{2}$ *Taking the square root of both sides*

$x + 3 = \pm\sqrt{2}$

Since 2 have two square roots (i.e., a positive and a negative square root), we have two linear equations:

- **Equation 1:** $x + 3 = \sqrt{2}$
- **Equation 2:** $x + 3 = -\sqrt{2}$

Step 6: Solve the resulting linear equations.

Solving each linear equation:

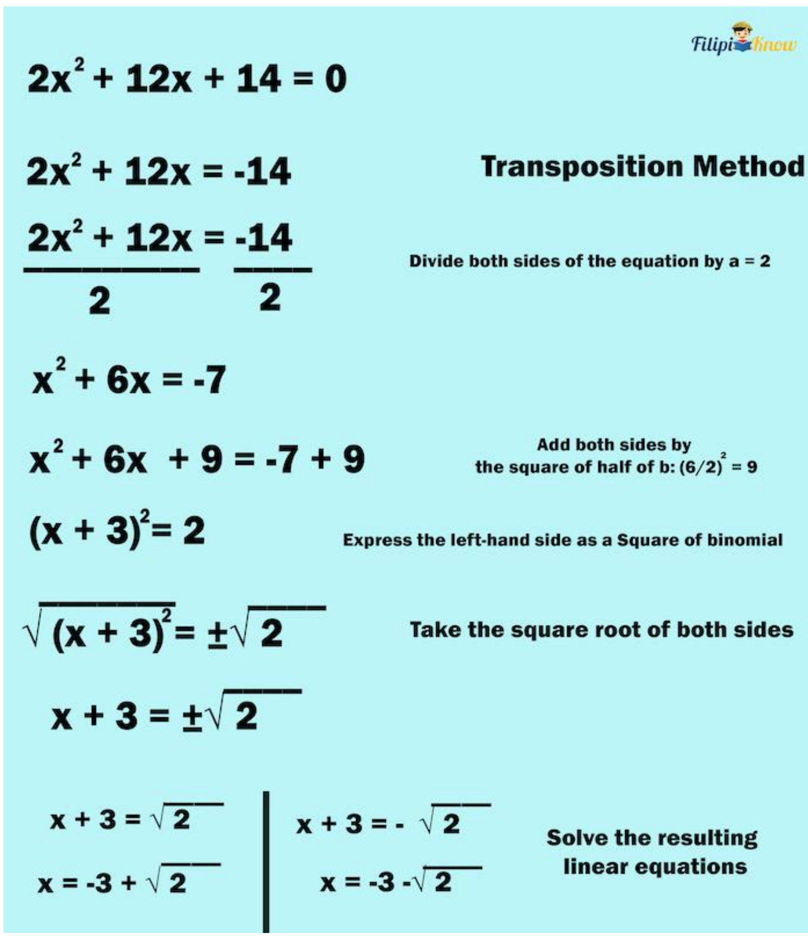
Equation 1: Equation 2:

$x + 3 = \sqrt{2}$ $x + 3 = -\sqrt{2}$

$x = -3 + \sqrt{2}$ $x = -3 -\sqrt{2}$

Therefore, the roots of the equation $2x^2 + 12x + 14 = 0$ are $\mathbf{x_1 = -3 + \sqrt{2}}$ **and** $\mathbf{x_2 = -3 -\sqrt{2}}$

Here's a quick review of what we have done above:



Method 4: How to Solve Quadratic Equation Using Quadratic Formula.

The quadratic formula is a formula that will give you the roots of a quadratic equation. You can use this formula for any type or form of quadratic equation.

The Quadratic Formula.

The quadratic formula is:

Quadratic Formula

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where:

- *a* is the numerical coefficient of the quadratic term
- *b* is the numerical coefficient of the linear term
- *c* is the constant term

The quadratic formula was derived by completing the square (see previous method). If you are curious how the quadratic formula was derived, kindly read the BONUS part of this reviewer.

To use the quadratic formula, you have to determine the values of *a, b,* and *c* first. Afterward, substitute these values to the formula and compute. You will arrive at two values because of the ± sign.

Example 1: *Use the quadratic formula to solve for the values of x in* $3x^2 - 2x - 1 = 0$

Solution:

The values of *a, b,* and *c* are:

- *a* = *3*
- *b* = - 2
- *c* = - *1*

Let us substitute these values to the quadratic formula:

$$x_1, x_2 = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)}$$

Computing for the values of *x*:

$$x_1, x_2 = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)}$$

$$x_1, x_2 = \frac{2 \pm \sqrt{4 - 4(3)(-1)}}{6}$$

$$x_1, x_2 = \frac{2 \pm \sqrt{4 + 12}}{6}$$

$$x_1, x_2 = \frac{2 \pm \sqrt{16}}{6}$$

$$x_1, x_2 = \frac{2 \pm 4}{6}$$

$$x_1 = \frac{2 + 4}{6} = \frac{6}{6} = 1$$

$$x_2 = \frac{2 - 4}{6} = \frac{-2}{6} = -\frac{1}{3}$$

Hence, the roots of the equation are **x_1 = 1 and x_2 = -⅓**

Example 2: *Use the quadratic formula to solve for the values of x in* $x^2 + 3x - 5 = 0$

The values of *a, b,* and *c* are:

- $a = 1$
- $b = 3$
- $c = -5$

Let us substitute these values to the quadratic formula:

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Downarrow$$

$$x_1, x_2 = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-5)}}{2(1)}$$

Computing for the values of *x*:

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_1, x_2 = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-5)}}{2(1)}$$

$$x_1, x_2 = \frac{-3 \pm \sqrt{9 - 4(1)(-5)}}{2}$$

$$x_1, x_2 = \frac{-3 \pm \sqrt{9 + 20}}{2}$$

$$x_1, x_2 = \frac{-3 \pm \sqrt{29}}{2}$$

$$x_1 = \frac{-3 + \sqrt{29}}{2}$$

$$x_2 = \frac{-3 - \sqrt{29}}{2}$$

Discriminant of a Quadratic Equation.

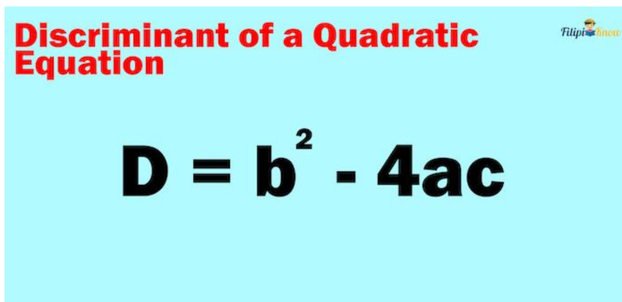
The **discriminant of a quadratic equation** allows you to determine the “nature of the roots” of a quadratic equation without actually solving it.

When we say “nature of the roots”, we are actually referring to three things:

1. The signs of the roots;
2. Whether the roots are real or complex numbers; and
3. Whether the roots are identical or not.

Through the discriminant, we can determine if a quadratic equation will give us roots that are real numbers or complex numbers, as well as if they are positive or negative numbers. We can also determine if the roots of that equation are identical or not.

The discriminant is the part of the quadratic formula that is under the radical sign (or the square root symbol).



What does the discriminant tell us?

- **If the computed value of the discriminant is positive ($D > 0$)**, then the quadratic equation has two real distinct roots (or two real different roots).
- **If the computed value of the discriminant is 0 ($D = 0$)**, then the quadratic equation has two identical real roots (or has only one root that is just repeated).
- **If the computed value of the discriminant is negative ($D < 0$)**, then the quadratic equation has no real roots. This means that the roots of the quadratic equation are complex numbers.

Let us have some examples:

Example 1: *Using the discriminant, determine the nature of the roots of* $x^2 + 4x + 4 = 0$

Solution:

We have $a = 1$, $b = 4$, and $c = 4$

Using the discriminant:

$D = b^2 - 4ac$

$D = (4)^2 - 4(1)(4)$

$D = 16 - 16$

$D = 0$

The value of the discriminant is 0. This means that the roots of $x^2 + 4x + 4 = 0$ are two identical roots (or one root that is just being repeated).

If we try to solve $x^2 + 4x + 4 = 0$ using factoring:

$x^2 + 4x + 4 = 0$

$(x + 2)(x + 2) = 0$ *by factoring*

$x_1 = -2 \quad x_2 = -2$

The roots of $x^2 + 4x + 4 = 0$ are both - 2. Indeed, the roots of the equation are identical and real numbers. The discriminant is correct.

Example 2: *What is the nature of the roots of* $2x^2 + 5x - 1 = 0$*?*

Solution:

We have $a = 2$, $b = 5$, and $c = -1$

Using the discriminant:

$D = b^2 - 4ac$

$D = (5)^2 - 4(2)(-1)$

$D = 25 + 8$

$D = 33$

The discriminant is positive. This means that the nature of the roots of $2x^2 + 5x - 1 = 0$ are real and distinct.

Try to verify using any method that the roots of $2x^2 + 5x - 1 = 0$ are distinct and real.

Example 3: *The quadratic equation $x^2 - (k + 2)x + 49 = 0$ has two identical roots. What must be the value of k?*

Solution:

If the quadratic equation has two identical roots, it means that its discriminant is equal to 0.

Thus, we set:

$D = b^2 - 4ac$

$0 = b^2 - 4ac$

We have $a = 1$, $b = -(k + 2)$, and $c = 49$. Substituting these values:

$0 = b^2 - 4ac$

$0 = (k + 2)^2 - 4(1)(49)$

Now, let us solve for the value of *k:*

$0 = k^2 + 4k + 4 - 196$ *Expanding the square of binomial*

$0 = k^2 + 4k - 192$

$k^2 + 4k - 192 = 0$ *Symmetric Property of Equality*

$(k + 16)(k - 12) = 0$ *by factoring*

$k + 16 = 0$ $k - 12 = 0$ *Equating each factor to 0*

$k = -16$ $k = 12$ *Solving for the values of k in the linear equations*

$k = - 16$ $k = 12$

Therefore, the values of *k* should be $\mathbf{k_1 = - 16}$ **and** $\mathbf{k_2 = 12}$

Sum and Product of the Roots of a Quadratic Equation.

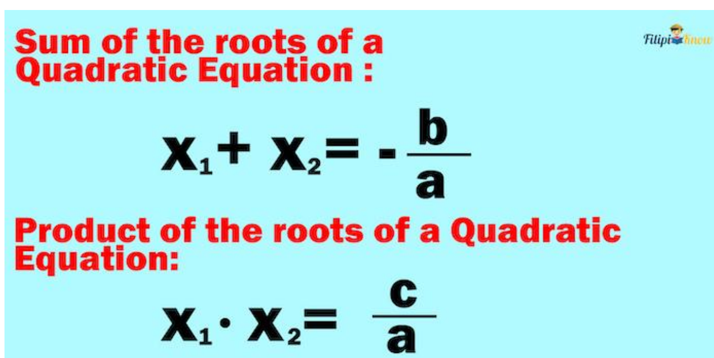
There's another fun thing about quadratic equations. We can determine the sum and the product of their roots without actually computing for them. All you have to do is to use the formulas below.

The formula for the sum of the roots of a quadratic equation is:

$x_1 + x_2 = -b/a$

The formula for the product of the roots of a quadratic equation is:

$x_1 \cdot x_2 = c/a$



The formulas above were derived using the quadratic formula but we will not show the derivation in this reviewer. You may refer here if you're interested in the formulas' derivation.

Example 1: *What is the sum and product of the roots of* $x^2 + 8x + 2 = 0$*?*

Solution:

Using the formulas for the sum and product of the roots:

Sum of the roots

$$x_1 + x_2 = -\frac{b}{a} = -\frac{b}{a} \quad \frac{-8}{1} = -8$$

Product of the roots

$$x_1 \cdot x_2 = \frac{c}{a} = \frac{2}{1} = 2$$

Thus, the sum and product of the roots are **- 8 and 2**, respectively.

Using Quadratic Equations to Solve Word Problems.

A lot of word problems can be described using quadratic equations. In this section, we're going to solve some word problems using quadratic equations.

Here are the steps that you may follow to solve word problems involving quadratic equations:

1. Read and understand the problem. Identify what is being asked.
2. Represent unknown quantities using a variable.
3. Construct a quadratic equation that represents the given situation or problem.
4. Solve the quadratic equation.

Example 1: *The sum of two numbers is 8 while their product is 15. What are the numbers?*

Solution:

Step 1: Read and understand the problem. Identify what is being asked. The problem is asking us to determine two numbers whose sum is 8 and whose product is 15.

Step 2: Represent unknown quantities using a variable. Let *x* be one of the numbers. Since the sum of the numbers is 8, we can represent the second number as 8 - *x*.

- **x** = first number
- **8 - x** = second number

Step 3: Construct a quadratic equation that represents the given situation or problem.
The problem states that the product of the numbers is 15. We can express it as:

(first number)(second number) = 15

Using the variables we set in Step 2:

$x (8 - x) = 15$

By the distributive property, we have:

$8x - x^2 = 15$

The equation above can be expressed in standard form as:

$-x^2 + 8x = 15$

Multiplying both sides by - 1, we obtain:

$x^2 - 8x = - 15$

$x^2 - 8x + 15 = 0$

Step 4: Solve the quadratic equation.

Let us solve for the values of x in $x^2 - 8x + 15 = 0$. We can solve this quadratic equation by factoring:

$x^2 - 8x + 15 = 0$

$(x - 3)(x - 5) = 0$

$x_1 = 3$ $x_2 = 5$

Based on the solution of our quadratic equation, the numbers are **3 and 5**.

Example 2: *A small rectangular garden has a perimeter of 12 meters and an area of 8 square meters. Determine the length and the width of the rectangular garden*.

Solution:

Step 1: Read and understand the problem. Identify what is being asked. The problem is asking us to determine the length and the width of the small rectangular garden given that its perimeter is 12 meters and its area is 8 square meters.

Step 2: Represent unknown quantities using a variable. Let l be the length of the small rectangular garden while let w represent its width.

Step 3: Construct a quadratic equation that represents the given situation or problem. The perimeter of a rectangle is defined as $P = 2l + 2w$. Since the small rectangular garden has a perimeter of 12 meters, we have:

$12 = 2l + 2w$ (Equation 1)

Meanwhile, the area of a rectangle is defined as $A = l \cdot w$. Since the small rectangular garden has an area of 8 square meters, we have:

$8 = l \cdot w$ (Equation 2)

Using Equation 1, we can solve for the value of w in terms of l:

$12 = 2l + 2w$

$12 = 2(l + w)$ *Distributive Property*

$6 = l + w$ *Division Property of Equality*

$w = 6 - l$ *Transposition Method*

Now, we substitute the value of w in terms of l in our second equation:

$8 = l \cdot w$

$8 = l(6 - l)$ *Substituting $w = 6 - l$*

$8 = 6l - l^2$

We have obtained the quadratic equation $8 = 6l - l^2$

Note that we can write it in standard form:

$8 = 6l - l^2$

$l^2 - 6l + 8 = 0$ *Transposition Method*

Step 4: Solve the quadratic equation.

Let us solve for l in $l^2 - 6l + 8 = 0$ by factoring:

$l^2 - 6l + 8 = 0$

$(l - 4)(l - 2) = 0$

$l_1 = 4$ $l_2 = 2$

Thus, we have two values for the length of the small rectangular garden according to our solution which are **4 meters and 2 meters.**

Let us substitute this value of l to $w = 6 - l$ to determine the possible values of the width:

Using $l_1 = 4$:

$w = 6 - 4$

$w_1 = 2$

Using $l_2 = 2$

$w = 6 - 2$

$w_2 = 4$

Thus, the possible values of the width are 2 meters and 4 meters.

Hence the values of the length and width of the rectangle are **4 meters and 2 meters**, respectively.

BONUS: How Was the Quadratic Formula Derived?

The quadratic formula was derived using the *completing the square* method:

$ax^2 + bx + c = 0$	*Standard form of a Quadratic Equation*
$ax^2 + bx = -c$	*Transposition Method*
$\frac{ax^2 + bx}{a} = \frac{-c}{a}$	*Dividing both sides of the equation by a*
$x^2 + \frac{b}{a}x = \frac{-c}{a}$	*Simplifying*
$x^2 + \frac{b}{a}x + \frac{b^2}{4a} = \frac{-c}{a} + \frac{b^2}{4a}$	*Adding both sides of the equation by the square of the half of* $\frac{b}{a}$
$(x + \frac{b}{2a})^2 = \frac{-c}{a} + \frac{b^2}{4a}$	*Expressing the left-hand side as a square of binomial*
$\sqrt{(x + \frac{b}{2a})^2} = \sqrt{\frac{-c}{a} + \frac{b^2}{4a}}$	*Taking the Square root of both sides*
$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$	*Simplifying*
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$	*Transposition Method*
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	*Simplifying*