Radical Expressions

Answer Key

1) Answer: A

Explanation: The index of the given radical $\sqrt[3]{x^4y^5}$ is 3. So, we think of the factors of x^4 and y^5 with an exponent of 3.

Note that x^3 is a factor of x^4 since $x^3 \bullet x = x^4$

Also, note that y^3 is a factor of y^5 since $y^3 \bullet y^2 = y^5$

This means that we can rewrite $\sqrt[3]{x^4y^5}$ as $\sqrt[3]{x^3 \bullet x \bullet y^3 \bullet y^2}$

The second property of radicals states that we can express root of a product of given quantities as product of the roots of the quantities:

$$\sqrt[3]{x^3 \bullet x \bullet y^3 \bullet y^2} = \sqrt[3]{x^3} \bullet \sqrt[3]{x} \bullet \sqrt[3]{y^3} \bullet \sqrt[3]{y^2}$$

Now, we can cancel out the radical sign of those radicals that have equal index and power of radicand. Therefore, we have:

$$\sqrt[3]{x^3} \bullet \sqrt[3]{x} \bullet \sqrt[3]{y^3} \bullet \sqrt[3]{y^2} = x \bullet \sqrt[3]{x} \bullet y \bullet \sqrt[3]{y^2} = xy\sqrt[3]{xy^2}$$

$xy\sqrt[3]{xy^2}$ is the simplified form of the given radical.

2) Answer: D

Explanation: The given expression in A is equivalent to $\sqrt{32}$ since $32^{1/2}$ is just $\sqrt{32}$ written as an expression with fractional exponent.

The given expression in B is also equivalent to $\sqrt{32}$. We know that $\sqrt{32} = 32^{1/2}$. The fractional exponent ½ is an equivalent fraction of 2/4. Hence, $32^{2/4}$ is also equivalent to $\sqrt{32}$.

The given expression in C is also equivalent to $\sqrt{32}$. Note that 16 x 2 is equal to 32. Thus, we can express $\sqrt{32}$ as $\sqrt{16\ x\ 2}$ which when simplified will give us $4\sqrt{2}$. Now $\sqrt{2}$ when transformed into an expression with a fractional exponent is $2^{1/2}$. This follows that $4\sqrt{2}$ is equal to $4(2)^{½}$

Since A to C provides values that are equivalent to $\sqrt{32}$. Then, it is logical to conclude that the expression in D is the one not equivalent to $\sqrt{32}$.

3) Answer: B

Explanation: The given radicals are like radicals so we can just combine them (i.e. add the coefficients and copy the common radical).

$4\sqrt{5} + 7\sqrt{5} = (4 + 7)\sqrt{5} = 11\sqrt{5}.$

The answer is $11\sqrt{5}$.

4) Answer: C

Explanation: To solve radical equations, we have to first isolate the expressions that are under the radical sign from those that are not under the radical sign. However, in the given radical equation $\sqrt{x - 2} = 5$, the terms under the radical sign are already isolated from other quantities (notice that the terms on the left-hand side are only those that are under the radical sign).

So, we can now raise both sides of the radical equation to the power equivalent to the index of the radical so that the radical sign will be removed. Since the index is 2, we raise both sides of the equation by 2:

$(\sqrt{x - 2})^2 = (5)^2$ — *raise both sides of the equation to the power of 2*

x - 2 = 25

x = 2 + 25 — *transposition method*

x = 27

Thus, the solution to the equation is 27.

5) Answer: B

Explanation To rationalize the denominator (or remove the radical sign in the denominator), we multiply the numerator and the denominator of the given expression by a radical so that the radical in the denominator will be eradicated.

The denominator of $\frac{1}{2\sqrt{x}}$ is $2\sqrt{x}$. Hence, we need to multiply $2\sqrt{x}$ by a certain radical that will make it into an expression that has no radical sign.

Note that if we multiply $\sqrt{x}$ to $2\sqrt{x}$, we will obtain $2\sqrt{x^2}$. We can remove the radical sign of $\sqrt{x^2}$ since it has equal index and power of radicand (both are 2).

This means that we should multiply the numerator and the denominator of $\frac{1}{2\sqrt{x}}$ by $\sqrt{x}$:

$$\frac{1 \bullet \sqrt{x}}{2\sqrt{x} \bullet \sqrt{x}} = \frac{\sqrt{x}}{2\sqrt{x^2}} = \frac{\sqrt{x}}{2x}$$

Based on our computation, the answer is $\frac{\sqrt{x}}{2x}$.