Factorials, Permutations, and Combinations

Answer Key

1) Answer: B
Explanation: We need to determine what number will have a factorial value equal to 5,040.

We can start with 3! since it is pretty easy to evaluate. Note that 3! = 3 x 2 x 1 = 6.

To obtain 4!, we just multiply 4 to 3!: 4 x 3! = 4 x 6 = 24.

Doing the same thing for 5!: 5 x 4! = 5 x 24 = 120

For 6!: 6 x 5! = 6 x 120 = 720

For 7!: 7 x 6! = 7 x 720 = 5,040

That's it, we have already obtained 5,040. We have discovered that 7! = 5040. Thus, the answer is x = 7.

2) Answer: A
Explanation: The number of distinguishable permutations can be calculated using the formula:

$$P_{\text{Distinguishable}} = \frac{n!}{p!\,q!\,\ldots}$$

where *n* is the total number of elements while *p* and *q* are the number of times an element is repeated.

FACTORIALS have 10 letters. So we have n = 10.
Note that A is repeated twice, so we expect to have 2! in the denominator.

Using the formula for distinguishable permutation:

$$\frac{10!}{2!} = \frac{10\,x\,9\,x\,8\,x\,7\,x\,6\,x\,5\,x\,4\,x\,3\,x\,2\,x\,1}{2\,x\,1} = 10\,x\,9\,x\,8\,x\,7\,x\,6\,x\,5\,x\,4\,x\,3 = 1,814,400$$

Thus, the answer is 1,814,400 distinguishable permutations.

3) Answer: B

Explanation: We set three blanks below to represent the three-digit numbers that can be formed using 0, 3, 4, 7, and 9:

_____ _____ _____

For the first digit (represented by the first blank), there are four numbers as options. Note that we cannot put 0 as the first digit since it will make the number two-digit only. Hence, we put 4 in the first blank:

4 _____ _____

Meanwhile, for the second blank, we can use four numbers as options. Note that we have already used one number for the first digit, and we can now use 0 for the second digit.

4 4 _____

Lastly, there are 3 options for the third blank since we have already used two numbers for the first two blanks:

4 4 3

Applying the fundamental counting principle:

$$4 \times 4 \times 3 = 48$$

Thus, the answer is 48.

4) Answer: B

Explanation: This problem is a combination case since the order of the elements does not matter when forming groups of people.

Since we are forming a group with 5 people from 12 people, we have n = 12 and r = 5.

Using the formula for combination:

$$C(n, r) = \frac{n!}{r!(n-r)!}$$

$$C(12, 5) = \frac{12!}{5!\,(12-5)!} = \frac{12!}{5!\,7!}$$

$$C(12, 5) = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

Note that we can cancel 7 x 6 x 5 x .. x 1 both in the numerator and in the denominator:

$$C(12, 5) = \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1}$$

Canceling out common factors:

$$\frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1}$$

$$\frac{12 \times 11 \times 10 \times 9 \times 8}{(4 \times 3) \times (5 \times 2) \times 1}$$

$$\frac{\cancel{12} \times 11 \times \cancel{10} \times 9 \times 8}{\cancel{(4 \times 3)} \times \cancel{(5 \times 2)} \times 1}$$

$$\frac{11 \times 9 \times 8}{1} = 792$$

Thus, the answer is 792.

5) Answer: D
Explanation: This problem is a case of combination since the order of the points that form a triangle does not matter.

We have n = 5 since there are 5 points on the plane and r = 3 since 3 points determine a triangle.

Using the formula for combination:

$$C(n, r) = \frac{n!}{r!(n-r)!}$$
$$C(5, 3) = \frac{5!}{3!(5-3)!}$$
$$C(5, 3) = \frac{5!}{3!\,2!} = \frac{5 \, x \, 4 \, x \, 3 \, x \, 2 \, x \, 1}{(3 \, x \, 2 \, x \, 1) \, x \, (2 \, x \, 1)}$$

We can cancel out 3 x 2 x 1 both in the numerator and in the denominator:

$$C(5, 3) = \frac{5 \, x \, 4}{2 \, x \, 1} = 20/2 = 10$$

Thus, the answer is 10 triangles.