Algebra is like solving a puzzle; one that challenges you to find a missing piece. In the case of algebra, that missing piece comes in the form of an unknown value.

The previous chapters of this algebra review focused on how to compute algebraic expressions, but we haven't started the "puzzle-solving" part of it yet.

Solving equations is the "puzzle-solving" part of algebra. In this chapter, you'll learn how to find the value of an unknown variable of a linear equation, just like looking for a missing piece in a puzzle.

What is an equation?

An **equation** is a mathematical statement that tells you that two quantities are equal in value.

To determine whether a mathematical statement is an equation or not, look for the equal sign (=). If there's a presence of the equal sign, then the mathematical statement is an equation.

For instance, 3 + 3 = 6 is an equation because it has an equal sign. 3 + 3 = 6 tells us that the value of 3 + 3 is similar to the value of 6.

Example: *Which of the following are equations?*

a. 2x + 3 = - 9

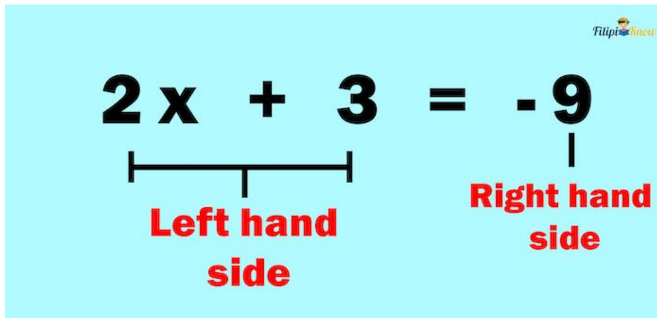
b. x = - 7

c. x - 5

Solution: The mathematical statements in *a* and *b* are equations because they have an equal sign. On the other hand, *c* is not an equation because of the absence of an equal sign.

In an equation, it is important to recognize its **left-hand side and its right-hand side.**

- **The left-hand side of the equation** - quantities on the left of the equal sign.
- **The right-hand side of the equation** - quantities on the right of the equal sign.



In the above example, the left-hand side of the equation *2x + 3 = -9* is *2x + 3* while its right-hand side is *-9*.

Solution to an Equation.

An equation involves a variable or a value that is unknown or not determined yet. When we say "solve an equation," what we mean is to determine the value being represented by that unknown variable to make the equation hold.

For example, x + 9 = 10 is an equation telling us that x + 9 must be equivalent to 10.

x is the unknown variable in the equation. When we solve for x + 9 = 10, we determine the value of *x* so that x + 9 will be equal to 10.

If x = 1, the left-hand side of the equation and the right-hand side of the equation will be of the same value.

If $x = 1$

$$x + 9 = 10$$
$$(1) + 9 = 10$$
$$10 = 10$$

Once we have shown that the left-hand side and the right-hand side of the equation are equal, then the value of the variable we used is the solution to the equation. Therefore, the solution to the equation x + 9 = 10 is **x = 1**.

On the other hand, let's say we use x = 2 for x + 9 = 10

If $x = 2$

$$x + 9 = 10$$
$$(2) + 9 = 10$$
$$11 \neq 10$$

In this case, the left-hand side and the right-hand side are not equal. Thus, x = 2 is not the solution to the equation x + 9 = 10.

Therefore, **the solution to an equation is the value of the unknown variable that will make the equation true. When we say that the equation is true, it means that the left-hand side and the right-hand side of it are equal in value.**

Example: *Is x = 5 the solution to x + 2 = 7?*

Solution: Yes, because if we substitute x = 5 to x + 2 = 7:

x + 2 = 7

(5) + 2 = 7

7 = 7

The left-hand side and the right-hand side of the equation are equal. Indeed, x = 5 is the solution to x + 2 = 7.

Come to think of it, an equation is a puzzle with a missing piece. That missing piece is the unknown variable. When you solve for the value of the unknown variable, you are actually looking for the missing piece that will complete the puzzle or the equation.

But how do we find that missing piece? How do we find the solution to the equation?

The answer is we apply the properties of equality to solve an equation. In the next section of this reviewer, we will be discussing these properties.

Properties of Equality.

The **properties of equality** are rules or principles that allow us to manipulate equations so we can determine the values of the unknown variable. We can use the properties of equality as the logical explanation for why we manipulate an equation in a certain way.

Here are the properties of equality:

1. Reflexive Property of Equality.

For any real number p:

$$p = p$$

This property is pretty obvious and logical. The value of a number is always equal to itself.

For instance, 1020 will always be equal to 1020. If someone tells you that 1020 = 1100, he is logically false since 1020 is always equal to 1020 by the reflexive property.

2. Symmetric Property of Equality.

For any real numbers p and q:

If p = q, then q = p

This property tells us that in an equation if we switch the positions of the quantities on the left-hand side and the right-hand side of the equation, the equation will still hold. This also implies that both sides of the equation are of the same value.

For example, we know that 3 + 4 = 1 + 6 is true. By the symmetric property of equality, 1 + 6 = 3 + 4 must also be true.

3. Transitive Property of Equality.

For any real numbers p, q, and r:

If p = q and q = r, then p = r

The transitive property of equality tells us that if a quantity is equal to a second quantity, and if the second quantity is equal to a third quantity, then we can conclude that the first quantity is equal to the third quantity.

For example, if we assume that x = y and y = w, then by the transitive property, we can conclude that x = w.

Another example: We know that 10 - 5 = 2 + 3 is true. We also know that 2 + 3 = 9 - 4. By the transitive property, we can conclude that 10 - 5 = 9 - 4.

4. Addition Property of Equality (APE).

For any real numbers p, q, and r:

If p = q, then p + r = q + r

APE tells us that if we add a certain number to two equal quantities, the result will still be equal.

For example, we know that 5 + 2 = 6 + 1 is true. Suppose that we add 8 to both sides of the equation:

$$5 + 2 = 6 + 1$$
$$5 + 2 + 8 = 6 + 1 + 8$$
$$15 = 15$$

Notice that even after adding the same number to both quantities, the resulting quantities are still equal.

APE implies that adding the same number to two equal quantities retains their equality.

5. Subtraction Property of Equality (SPE).

For any real numbers p, q, and r:

If $p = q$, then $p - r = q - r$

What if we subtract the same number to two equal quantities? Will equality be retained?

If we subtract a number from two equal quantities, the equality will still be retained.

For instance, we know that 5 + 2 = 6 + 1 is true. Suppose that we subtract 2 from both sides of the equation:

$$5 + 2 = 6 + 1$$
$$5 + 2 - 2 = 6 + 1 - 2$$
$$5 = 5$$

As we can see above, equality is retained.

SPE tells us that if we subtract two equal quantities by the same number, the results will still be equal.

6. Multiplication Property of Equality (MPE).

For any real numbers p, q, and r:

If p = q, then pr = qr

MPE tells us that the results will still be equal if we multiply two equal quantities by the same number.

For example, we know that 2 + 2 = 3 + 1. Suppose that we multiply both sides of this equation by 5:

$$2 + 2 = 3 + 1$$
$$5 \cdot (2 + 2) = 5 \cdot (3 + 1)$$
$$5 \cdot 4 = 5 \cdot 4$$
$$20 = 20$$

As shown above, the results will still be equal even after multiplying both sides by the same number.

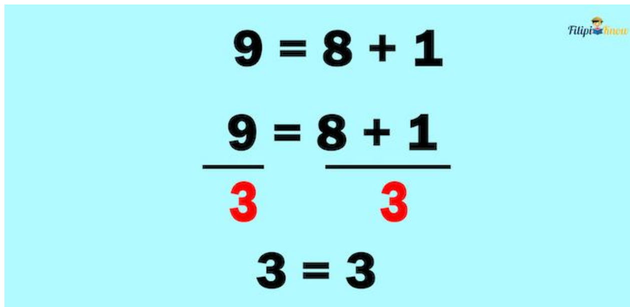
7. Division Property of Equality.

For any real numbers p, q, and r where r ≠ 0:

If p = q, then p/r = q/r

This property tells us that the results will still be equal if we divide two equal quantities by the same number (that number can be any number but must not be equal to 0).

For example, we know that 9 = 8 + 1. Suppose that we divide both sides of this equation by 3:



As per the division property of equality, the results are still equal.

8. Distributive Property of Equality.

For any real numbers p, q, and r:

$$p(q + r) = pq + pr$$

You most likely remember learning about this in the previous reviewer (i.e., multiplication of polynomials). This property tells us that multiplying the sum of two or more addends is equal to the result when we multiply the addends by that number and add them.

For example, suppose that we want to double the sum of 3 and 5. We can express it as:

$$2(3 + 5)$$

By the distributive property, we can distribute 2 to each addend and still preserve equality.

$$2(3) + 2(5)$$

Now, 2(3) + 2(5) = 16. Hence, **2(3 + 5) = 2(3) + 2(5) = 16**

9. Substitution Property of Equality.

If x = y, then either x or y can be substituted into any equation for the other.

Suppose that x + y = 12. If we assume that x = y, then we can replace *y* with *x* and the equation will still hold.

Thus, if x = y, then x + y = 12 can be x + x = 12 or y + y = 12.

Here's a table that summarizes the properties of equality:

Property of Equality *(Suppose that p, q, and r are real numbers)*	**Summary**
Reflexive Property	*p = p*
Symmetric Property	*If p = q, then q = p*
Transitive Property	*If p = q and q = r, then p = r*
Addition Property of Equality	*If p = q, then p + r = q + r*
Subtraction Property of Equality	*If p = q, then p - r = q - r*
Multiplication Property of Equality	*If p = q, then pr = qr*
Division Property of Equality	*If p = q, then p/r = q/r where r 0*
Distributive Property of Equality	*p(q + r) = pq + pr*
Substitution Property of Equality	*If x = y and ax + by = c, then ax + bx = c or ay + by = c*

We will use the properties above to manipulate equations and determine the value of an unknown variable. In other words, these properties will be used to find the solution of an equation.

Linear Equations in One Variable.

The first type of equation that we will learn how to solve is linear equations in one variable. These equations are the simplest type of equations and the easiest ones to answer.

Linear equations are equations such that the highest exponent of its variable is 1. The type of linear equations that we are going to solve in this section is those with one variable only (linear equations with more than one variable will be discussed in the later sections).

For example, x + 3 = 9 is a linear equation since the highest exponent of its variable is 1. As you can notice, x + 3 = 9 has only one variable involved (which is *x*). Thus, x + 3 = 9 is **a linear equation in one variable.**

In other words, linear equations in one variable are in the form *ax* + *b* = *c*, where *a*, *b,* and *c* are real numbers and *a* ≠ *0*.

Example: *Which of the following are linear equations in one variable?*

a. 2x + 7 = 19

b. $x^2 + 6x + 9 = 0$

c. x + y = 2

Solution: The equation in *a* is the only linear equation in one variable among the given equations since the highest exponent of its variable is 1 and it has only one variable. The equation in *b* is not a linear equation since the highest exponent of its variable is 2 (it is a quadratic equation). Meanwhile, the equation in c, although the highest exponent of its variable is 1, is not a linear equation in one variable because there are two variables involved (i.e., *x* and *y*).

Furthermore, take note that the equation in *a* is the only equation in the form *ax* + *b* = *c* form.

In the next section, you are going to learn how to solve linear equations in one variable by applying the properties of equality that we have discussed above.

How to Solve Linear Equations in One Variable.

Example 1: *Let us try to solve for the value of x in x - 9 = 10.*

Solution: To find the value of *x*, our goal is to isolate the variable from the constants. This means that if we want to solve for *x*, then *x* must be the only quantity on the left side of the equation and the other quantities must be on the right side. *But how can we achieve that?*

x will be the only quantity on the left if we get rid of -9 on the left side. *How can then we remove -9 on the left side?*

The addition property of equality (APE) states that we can add the same number to both sides of the equation.

Applying the APE, we can add 9 to both sides of the equation so we can cancel -9 on the left side:

$$x - 9 = 10$$

$$x - 9 + 9 = 10 + 9$$

$$x = 19$$

Now, it is seen that x = 19.

That's it! We have solved the value of *x* in x - 9 = 10. The answer is **x = 19**.

Example 2: *Solve for x in x - 12 = 22*

Solution:

x - 12 = 22

x - 12 + 12 = 22 + 12 *(adding 12 to both sides of the equation)*

x = 34

Hence, **x = 34**.

Note that the explanation for why it is valid to add 12 to both sides of the equation is that we apply the addition property of equality.

Example 3: *Solve for x in x + 10 = 52*

Solution: To isolate *x* from the constants, we must get rid of 10 by subtracting 10 from both sides of the equation. Subtracting the same number from both sides of the equation is valid because of the subtraction property of equality (SPE) discussed earlier.

$x + 10 = 52$

$x + 10 - 10 = 52 - 10$ *(subtracting 10 from both sides of the equation)*

$x = 42$

Thus, the answer is **x = 42**

Transposition Method.

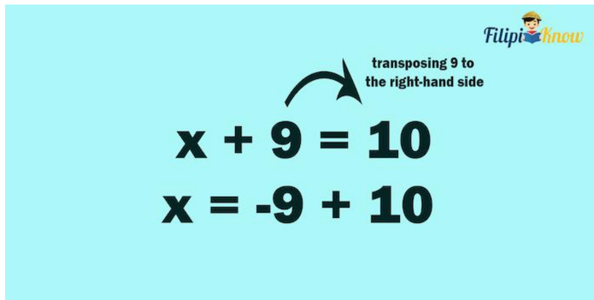
There is a “shortcut” method that we can use instead of applying the APE or SPE. To isolate *x* from the constants, we can transpose the constant to the right-hand side of the equation so that *x* will be the only quantity that will remain on the left side.

Example 1: *Let us solve x + 9 = 10 using the transposition method.*

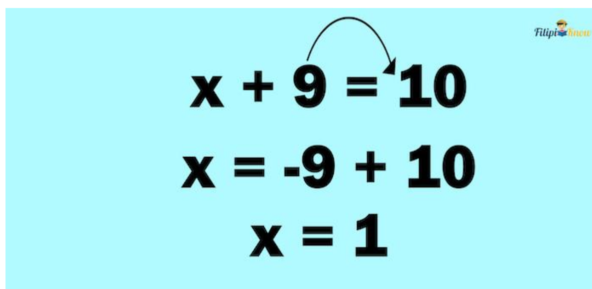
Solution:

Our goal is to isolate *x* from other constants by transposing 9 to the right-hand side of the equation. Once a quantity “crosses” the equality sign, its sign reverses (i.e., from positive 9 to -9).

After we transpose 9 to the right-hand side and reverse its sign, we add it to the quantity on the right-hand side (which is 10).



Then, we perform some arithmetic:



Thus, the answer is **x = 1.**

Example 2: *Use the transposition method to solve for x in x - 9 = 12.*

Solution: Transposing -9 to the right-hand side will reverse its sign (i.e., from negative to positive):

x = 9 + 12

x = 21

Thus, the answer is **x = 21.**

Example 3: *Solve for x in x + 6 = 5 using the transposition method.*

Solution: Transposing 6 to the right-hand side will reverse its sign (i.e., from positive to negative):

$x = -6 + 5$

$x = -1$

Thus, the answer is **x = -1**.

Example 4: *Solve for x in x - 4 = - 9 using the transposition method.*

Solution:

$x - 4 = -9$

$x = 4 + (-9)$ *(transposing -4 to the right-hand side will change its sign to positive)*

$x = -5$

Thus, the answer is **x = -5.**

Note: In this reviewer, we will be using the transposition method more frequently to isolate *x* from other quantities. The transposition method is more convenient than adding numbers to or subtracting numbers from both sides of the equation.

Applying the Division Property of Equality to Solve Linear Equations in One Variable.

Most of the linear equations in one variable we have solved above are in the form of $ax + b = c$ where $a = 1$ (the coefficient of *x* is 1) But what if *a* is not equal to 1 like in $2x + 4 = 6$? If this is the case, we can solve for x by applying the division property of equality.

Example 1: *Let us try to solve for x in 2x + 4 = 6.*

Solution: Again, to solve for *x* in an equation, it must be isolated from the constants or *x* should be the only quantity on the left-hand side of the equation.

Let us start by getting rid of 4 on the left-hand side by using the transposition method:

$$2x + 4 = 6$$

$$2x = -4 + 6$$ **Transposition method**

$$2x = 2$$

What is left is 2x = 2. Again, our goal is to make *x* the only quantity on the left-hand side. This means that we need to cancel out 2 in 2x. *But how do we cancel it?*

We can divide both sides of the equation by 2 so that 2 will be canceled in 2x. This is valid because the division property of equality guarantees us that dividing both sides of the equation by the same number will preserve equality.

$$2x = 2$$

$$\frac{2x}{2} = \frac{2}{2}$$ **Dividing both sides by 2**

$$x = 1$$

As we can see, the answer is **x = 1.**

Here’s a quick preview of what we have done above:

2x + 4 = 6

2x = -4 + 6 *(transposing 4 to the right-hand side will turn it into -4*)

2x = 2

2x⁄2 = 2⁄2 *(dividing both sides of the equation by 2)* x = 1

Thus, the solution to 2x + 4 = 6 is **x = 1**

You can verify that x = 1 is the solution by substituting it back to 2x + 4 = 6. Notice that the equation will be true if x = 1:

2(1) + 4 = 6

2 + 4 = 6

6 = 6

Example 2: *Solve for x in 3x - 18 = 27*

Solution: To solve for *x*, *x* should be the only quantity on the left-hand side.

We start by transposing -18 to the right-hand side. If we transpose it, it will have a positive sign.

3x = 18 + 27

3x = 45

To cancel out *3* in 3x, we divide both sides of the equation by 3:

3x⁄3= 45⁄3

x = 15

Therefore, the answer is **x = 15**

Example 3: *Solve for x in 4x - 18 = 2*

Solution: To solve for *x*, *x* should be the only quantity on the left-hand side.

We start by transposing -18 to the right-hand side. If we transpose it, it will change its sign from negative to positive.

4x = 18 + 2

4x = 20

To cancel out *4* in 4x, we divide both sides of the equation by 4:

$4x/4 = 20/4$

x = 5

Therefore, the answer is **x = 5**.

Example 4: *Solve for x in 7x + 2 = 16*

Solution:

7x + 2 = 16

7x = -2 + 16 *Transposition Method (we transpose 2 to the right-hand side)*

$7x/7 = 14/7$ *Division Property of Equality (divide both sides of the equation by 7)*

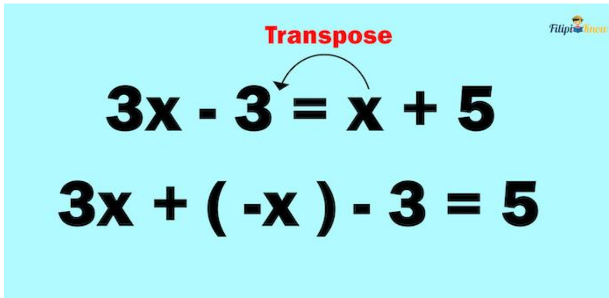
x = 2

More Examples of Solving Linear Equations in One Variable.

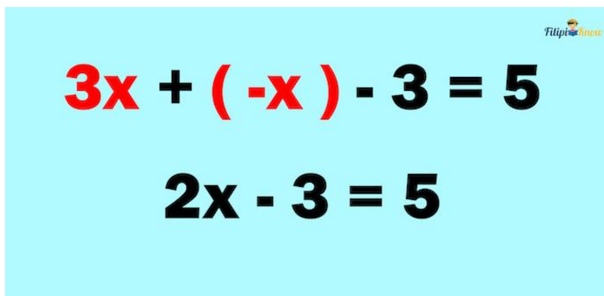
This section contains more linear equations in one variable to solve. However, these equations are trickier than what we have solved so far since they appear in different forms. Just keep in mind three things so you can solve them: the properties of equality, the transposition method, and our goal to isolate x from other constants (or x should be the only quantity on the left-hand side).

Example 1: *Solve for x in 3x - 3 = x + 5*

Solution: Let us put all *x* first on the left-hand side. We can do this by transposing the *x* on the right-hand side to the left-hand side. Like numbers, variables will also reverse their sign once they cross the equality sign.



We can then combine 3x and -x to obtain 2x:



Now, we have 2x - 3 = 5. We can apply the techniques we have learned above to solve this one:

$2x - 3 = 5$

$2x = 3 + 5$ *Transposition Method*

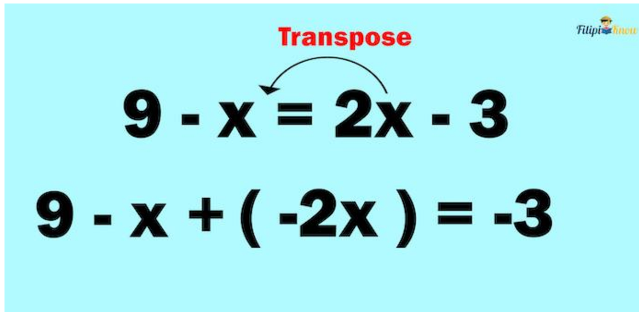
$2x = 8$

$2x/2 = 8/2$ *Division Property of Equality*

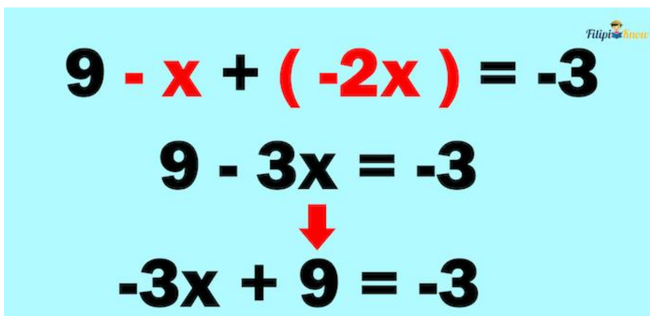
x = 4

Example 2: *Solve for 9 - x = 2x - 3*

Solution: We start by putting all *x* on the left-hand side of the equation using the transposition method.



We can then combine -2x and -x to obtain -3x:



Thus, we have -3x + 9 = - 3. Let us now use the techniques we have learned to solve for *x*:

-3x + 9 = -3

-3x = -9 + (-3) *Transposition Method*

-3x = -12

-3x⁄-3= 12⁄-3 *Division Property of Equality*

x = - 4

Example 3: *Solve for 3(2x + 1) = 15*

Solution: Since *3* is multiplied by the sum of addends, we can apply the distributive property so that our equation will be in the form *ax* + *b* = *c*.

$3(2x + 1) = 15$

$3(2x) + 3(1) = 15$ *Distribute 3 to 3(2x + 1)*

$6x + 3 = 15$

Now, let us continue the process using the techniques we have learned in the previous sections:

$6x = -3 + 15$ *Transposition Method*

$6x = 12$

$6x/6 = 12/6$ *Division Property of Equality*

x = 2

Therefore, the answer is **x = 2**

Example 4: *Solve for x in* $\frac{3x+2}{2} = \frac{1}{3}$

Solution: In case a linear equation in one variable is fractional in form, we "remove" the denominator by multiplying both sides of the equation by the Least Common Denominator (this method is valid because of the multiplication property of equality).

The Least Common Denominator (LCD) is the lowest common multiple of the denominators 3 and 2. Therefore, the LCD should be 6.

We then multiply both sides of the equation by the LCD (which is 6):

$$6\left(\frac{3x+2}{2}\right) = 6\left(\frac{1}{3}\right)$$

$$\frac{6}{2}(3x+2) = \frac{6}{3}$$

$$3(3x+2) = 2$$

Now, our equation becomes 3(3x + 2) = 2

Let us continue solving for *x*:

3(3x + 2) = 2	
3(3x) + 3(2) = 2	*Distributive Property*
9x + 6 = 2	
9x = -6 + 2	*Transposition Method*
9x = -4	
$\frac{9x}{9} = \frac{-4}{9}$	*Division Property of Equality*

x = -4/9

Therefore, the answer is **x = -4/9**

Example 5: *Solve for x in* $\frac{x+4}{2} = \frac{1}{4}$

Solution:

$\frac{x+4}{2} = \frac{1}{4}$	
$4(\frac{x+4}{2}) = 4(\frac{1}{4})$	*Multiply both sides of the equation by the LCD (which is 4)*
2(x + 4) = 1	
2(x) + 2(4) = 1	*Distributive Property*
2x + 8 = 1	
2x = -8 + 1	*Transposition Method*
2x = -7	
$\frac{2x}{2} = \frac{-7}{2}$	*Division Property of Equality*
$x = \frac{-7}{2}$	

Solving Word Problems Using Linear Equations in One Variable.

Now that you have learned the essential techniques and principles to solve linear equations in one variable, we can apply this skill to solve some word problems.

To solve word problems using linear equations, follow these steps:

1. Read and understand the given problem and determine what is being asked.
2. Represent the unknown in the problem using a variable.
3. Construct a linear equation that will describe the problem.
4. Solve for the value of the unknown variable in the linear equation.

Example 1: *The sum of a number and 5 is - 3. What is the number?*

Solution:

Step 1: Read and understand the given problem and determine what is being asked. The problem is asking us to determine the number such that the sum of that number and 5 is - 3.

Step 2: Represent the unknown in the problem using a variable. Let x represent the number we are looking for.

Step 3: Construct a linear equation that will describe the problem. The problem states that the sum of the unknown number (represented by x) and 5 is - 3. Therefore, we construct the linear equation below:

$x + 5 = -3$

Step 4: Solve for the value of the unknown variable in the linear equation. Using the equation we have derived from Step 3, we solve for the value of x:

$x + 5 = -3$

$x = -5 + (-3)$ *Transposition Method*

$\mathbf{x = -8}$

Thus, the number is -8.

Example 2: *Fred has 52 books in his collection. He gave some of these books to Claude. Fred also gave some books to Franz. The number of books that Fred gave to Franz is twice the number of books that he gave to Claude. The number of books left to Fred after he gave some to Claude and Franz is 22. How many books did Claude receive?*

Solution:

Step 1: Read and understand the given problem and determine what is being asked. The problem is asking us to determine the number of books Claude received from Fred.

Step 2: **Represent the unknown in the problem using a variable**. Let x be the number of books Claude received. Since Franz received twice the number of books that Claude received, we let $2x$ be the number of books Franz received.

To summarize:

- x = number of books that Claude received
- $2x$ = number of books that Franz received

Step 3: Construct a linear equation that will describe the problem. It's stated that after Fred gave some books to Claude and Franz, there were only 22 books left.

We can express this statement this way:

52 - (number of books that Claude received) - (number of books that Franz received) = 22

Using the variables we have set in Step 2:

$52 - x - 2x = 22$

Step 4: Solve for the value of the unknown variable in the linear equation.

$52 - x - 2x = 22$

$52 - 3x = 22$ *Combining like terms*

$-3x = -52 + 22$ *Transposition Method*

$-3x = -30$

$-3x/-3 = -30/-3$ *Division Property of Equality*

x = 10

Since *x* represents the number of books Claude received from Fred, then Claude received 10 books from Fred.

Using the value of *x* that we have obtained in the problem, *can you determine how many books Franz received from Fred?*

Yes, the answer is 20 since Franz received twice the number of books Claude received.

Example 3: *The total number of participants in a mini-concert by a local band is 300. The number of female participants in the mini-concert is half the number of male participants in the event. How many male participants are there in the mini-concert?*

Solution:

Step 1: Read and understand the given problem and determine what is being asked. The problem is asking us to determine the number of male participants in the mini-concert.

Step 2: Represent the unknown in the problem using a variable. Let *x* be the number of male participants in the mini-concert. Since the number of female participants in the mini-concert is half the number of male participants, we let ½ *x* represent the number of female participants in the event.

Step 3: Construct a linear equation that will describe the problem. The total number of participants in the mini-concert is 300. We can express this as:

(Number of Male Participants) + (Number of Female Participants) = 300

Using the variables we have set in Step 2:

$$x + \frac{1}{2}x = 300$$

Step 4: Solve for the value of the unknown variable in the linear equation.

Let us solve for x in $x + \frac{1}{2}x = 300$

$x + ½ x = 300$

$2(x + ½ x) = 2(300)$ *Multiplying both sides of the equation by the LCD*

$2(x) + 2(½ x) = 600$ *Distributive Property*

$2x + x = 600$

$3x = 600$

$3x/3 = 600/3$ *Division Property of Equality*

x = 200

Since *x* represents the number of male participants of the mini-concert, there are 200 male participants.

Linear Equations in Two Variables (Systems of Linear Equations).

As the name suggests, **linear equations in two variables** are linear equations with two variables involved. For instance, $x + y = 5$ is an example of a linear equation in two variables because there are two variables involved (i.e., *x* and *y*).

Formally, linear equations in two variables are in the form $ax + by = c$ where *a, b,* and *c* are real numbers and *a* and *b* are both nonzero.

Solutions of Linear Equations in Two Variables.

Linear equations in two variables have a pair of solutions--one for *x* and one for y. For example, one possible solution for $x + y = 5$ is $x = 2$ and $y = 3$.

However, take note that there are other pairs of *x* and *y* that will satisfy $x + y = 5$. For instance, if $x = 0$ and $y = 5$, the equation will be true. Also, if $x = 1$ and $y = 4$, the equation will also be true. In other words, there are infinite values of *x*and *y* that will satisfy $x + y = 5$!

A linear equation in two variables has infinite possible values of *x* and *y*. For this reason, we need another two or more linear equations in two variables that will provide us a single pair of values of *x* and *y* only.

Let us add x - y = 1 in the discussion. For instance, if we solve for the values of *x* and *y* that satisfy x + y = 5 and x - y = 1 at the same time, we obtain x = 3 and y = 2. Note that these values of *x* and *y* are the only values that will satisfy both x + y = 5 and x - y = 1.

If x = 3 and y = 2

Equation 1	*Equation 2*
x + y = 5	**x - y = 1**
(3) + (2) = 5	**(3) - (2) = 1**
5 = 5	**1 = 1**

The pair of equations x + y = 5 and x - y = 1 is called a system of linear equations.

A system of linear equations is composed of two or more linear equations. The solution of a system of linear equations will satisfy all of the equations in the system.

Again, the pair x + y = 5 and x - y = 1 is an example of a system of linear equations.

At x = 3 and y = 2, the equations are both satisfied:

x + y = 5

(3) + (2) = 5 at *x = 3 and y = 2*

5 = 5

x - y = 1

(3) - (2) = 1 at *x = 3 and y = 2*

1 = 1

Therefore, x = 3 and y = 2 is the solution of the system of linear equations x + y = 5 and x - y = 1.

How to Solve a System of Linear Equations.

There are different ways of solving a system of linear equations. In this section, we will discuss two methods: the substitution method and the elimination method.

1. How to Solve a System of Linear Equations by Substitution.

To solve a system of linear equations using the substitution method, follow these steps:

1. Solve for the value of one variable in one of the linear equations in terms of the other variable.
2. Substitute the expression for the variable you have obtained in Step 1 in the other linear equation.
3. Solve for the value of the other variable in the equation you have obtained from Step 2.
4. Plug in the value of the unknown variable you have computed in Step 3 in the expression you have obtained in Step 1 to find the value of the other variable.

The steps might be too abstract at this moment, but they are easy to follow. Let us use these steps in our example below:

Example 1: *Solve for the values of x and y that will satisfy x + y = 9 and x - y = 3*

Solution:

Let us write first the given equations:

Equation 1: *x + y = 9*

Equation 2: *x - y = 3*

Step 1: Solve for the value of one variable in one of the linear equations in terms of the other variable. Using Equation 1, we solve for the value of *y* in terms of *x*. This means we let *y* be the only quantity on the left-hand side while the other quantities must be on the right-hand side, including *x*. To make this possible, we just transpose *x* to the right side:

$$x + y = 9 \longrightarrow y = -x + 9$$

Step 2: Substitute the expression for the variable you have obtained in Step 1 in the other linear equation. We have obtained y = -x + 9 in Step 1. What we are going to do is to substitute this value of *y* into the *y* in Equation 2:

x - y = 3 (*Equation 2*)

x - (-x + 9) = 3 *(We substitute y = -x + 9)*

Notice that once we substitute y = -x + 9 in Equation 2, Equation 2 will now be a linear equation in one variable.

Step 3: Solve for the value of the other variable in the equation you have obtained from Step 2. The equation we have obtained in Step 2 is x - (-x + 9) = 3. Our goal now is to solve for *x*.

We just use the techniques in solving linear equations in one variable:

x - (-x + 9) = 3

x + x - 9 = 3 *Distributive Property*

2x - 9 = 3

2x = 9 + 3 *Transposition Method*

2x = 12

2x⁄2= 12⁄2 *Division Property of Equality*

x = 6

Now that we have obtained the value for *x* which is *x = 6*, let us solve for *y*.

Step 4: Plug in the value of the unknown variable you have computed in Step 3 in the expression you have obtained in Step 1 to find the value of the other variable. From Step 3, we have obtained x = 6. We substitute *x* to the equation we obtained in Step 1, y = -x + 9.

y = -x + 9 *(The expression we have obtained in Step 1)*

y = -(6) + 9 *(Substitute x = 6 which we have obtained in Step 3)*

y = 3

That's it! The solution for our system of linear equations is **x = 6 and y = 3.**

2. How to Solve a System of Linear Equations by Elimination.

To solve a system of linear equations using the elimination method, follow these steps:

1. Write the given equations in standard form.
2. Add or subtract the given equations so that one variable will be eliminated. If there's no variable that can be eliminated by adding or subtracting the equations, you may multiply an equation by a constant to allow the elimination of a variable.
3. Solve for the value of the remaining variable.
4. Substitute the value of the variable you have computed in Step 3 to any of the given equations then solve for the value of the other variable.

Let us follow the above steps in our example below:

Example 1: *Solve for the values of x and y that will satisfy x + y = 10 and x - y = 12*

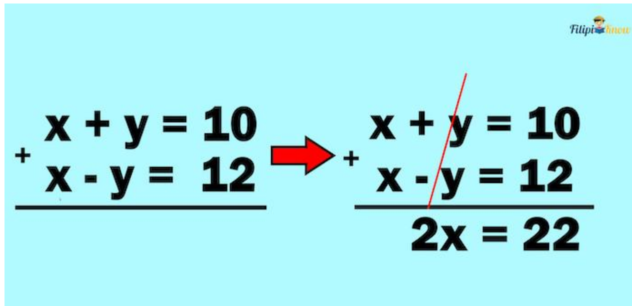
Solution:

Step 1: Write the given equations in standard form. If you can recall, the standard form of a linear equation in two variables is *ax* + *by* = *c*. Both x + y = 10 and x - y = 12 are already in standard form, so we can skip this step.

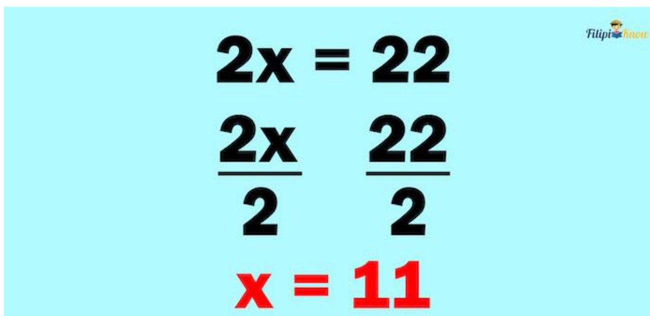
Step 2: **Add or subtract the given equations so that one variable will be eliminated. If there's no variable that can be eliminated by adding or subtracting the equations, you may multiply an equation by a constant to allow the elimination of a variable.**

If we add the equations x + y = 10 and x - y = 12, the *y* variable will be eliminated. There's no need to multiply the equations with a constant since we can immediately cancel a variable just by adding the equations.

After adding the equations, the resulting equation will be 2x = 22



Step 3: Solve for the value of the remaining variable. The remaining variable in 2x = 22 is *x*. We solve for *x* in this step by dividing both sides of 2x = 22 by 2:



Thus, x = 11

Step 4: Substitute the value of the variable you have computed in Step 3 to any of the given equations then solve for the value of the other variable. We substitute x = 11 to one of the given equations. Let us use x + y = 10:

x + y = 10

(11) + y = 10 *Substituting x = 11*

y = -11 + 10 *Transposition Method*

y = -1

Therefore, the solution for the system of linear equation is **x = 11 and y = -1.**