Logarithms

Answer Key

1) Answer: C

Explanation: We can simplify $\log_5 x + \log_5 y = z$ into a single logarithm before we can write it in exponential form.

As per the **Product Property of Logarithms**, the logarithm of a product can be expressed as the sum of logarithms (and vice versa). Note that $\log_5 x + \log_5 y$ is a sum of logarithms ($\log_5 x$ and $\log_5 y$).

Hence, we can express $\log_5 x + \log_5 y$ as the logarithm of the product of their arguments *x* and *y* to the base 5.

$\log_5 x + \log_5 y = z$
$\log_5 xy = z$ *Product Property of Logarithms*

Let us now express $\log_5 xy = z$ into fractional form by following these steps:

1) Drop the "log" sign

$$\cancel{\log}_5\, xy = z$$

2) Use the base of the logarithmic form (the small number on the lower right of "log" symbol) as the base of the exponential form

   The base in $\log_5 xy = z$ is 5. Hence, we use it as the base in the exponential form as well.

$$5^{\blacksquare} = \blacksquare$$

   The purple boxes serve as placeholders for the remaining elements in the exponential form.

3) Use the exponent of the logarithmic form (the number on the right of the equal sign) as the exponent for the exponential form

$$5^{z} = \blacksquare$$

4) Use the argument as the value of the logarithmic form

$$5^{z} = xy$$

Therefore, the equivalent exponential form of $\log_5 x + \log_5 y = z$ is $5^z = xy$.

2) Answer: A

Explanation: log 100, a common logarithm, has a base of 10. Hence, log 100 is $\log_{10}100$. To evaluate $\log_{10}100$, think of an exponent to which we must raise 10 so that we get 100.

Note that if we raise 10 to the power of 2, we will obtain 100 ($10^2 = 10 \times 10 = 100$).

This implies that $\log_{10}100 = 2$. So, the value of log 100 = 2.

To calculate the value of $\frac{1 - \log 100}{\log 100 + 1}$, we have to replace log 100 with 2:

$$\frac{1 - \log 100}{\log 100 + 1} = \frac{1 - 2}{2 + 1} = \frac{-1}{3}$$

Thus, the value of $\frac{1 - \log 100}{\log 100 + 1}$ is -⅓ .

3) Answer: B

Explanation: At first glance, $\log_a(2(x - y))^b$ involves an argument raised to an exponent (which is $(2(x - y))^b$). According to the **Power Property of Logarithms**, we can express the logarithm of a quantity raised to an exponent as the product of the exponent and the logarithm of the quantity.

Thus, by applying the Power Property, $\log_a(2(x - y))^b$ can be expressed as $b \times \log_a(2(x - y))$.

We can still expand $b \times \log_a(2(x - y))$; in particular, $\log_a(2(x - y))$.

$\log_a(2(x - y))$ has an argument that is the product of 2 and x - y. This implies that we can expand it by applying the **Product Property of Logarithms**. This property allows us to express the logarithm of a product as a sum of logarithms:

Therefore, by Product Property, we have $\log_a(2(x - y)) = \log_a 2 + \log_a(x - y)$.

Hence, $b \times \log_a(2(x - y)) = b \times (\log_a 2 + \log_a(x - y)) = b(\log_a 2 + \log_a(x - y))$.

Thus, the expanded form of $\log_a(2(x - y))^b$ is $b(\log_a 2 + \log_a(x - y))$.

Here's the summary of our computation above:

$log_a(2(x - y))^b$	
$b(log_a(2(x - y))$	*Power Property of Logarithms*
$b\ (log_a 2 + log_a(x - y))$	*Product Property of Logarithms*

4) Answer: C

Explanation: We need to determine the value of $(\log_a \frac{p}{q})^2$ given that $\log_a p = 3$ and $\log_a q = 4$.

For us to be able to use the given values $\log_a p = 3$ and $\log_a q = 4$, we have to expand $\log_a \frac{p}{q}$ first in the given expression $(\log_a \frac{p}{q})^2$?

It's important to remind you that the Power Property cannot be applied to $(\log_a \frac{p}{q})^2$ since the argument of the logarithm is not raised to the power of 2. Instead, it is the entire value of the logarithm that is raised to the aforementioned exponent.

This means that we are expanding $\log_a \frac{p}{q}$. Since we are dealing with an argument that is a quotient of two quantities (*p* and *q*), we should use the **Quotient Property**:

$$\log_a \frac{p}{q} = \log_a p - \log_a q$$

Hence, $(\log_a \frac{p}{q})^2$ is equivalent to $(\log_a p - \log_a q)^2$.

We can now use the given values $\log_a p = 3$ and $\log_a q = 4$:

$(\log_a \frac{p}{q})^2$
$(\log_a p - \log_a q)^2$ *Quotient Property of Logarithms*
$(3 - 4)^2$ *By substitution*
$(-1)^2 = 1$

Hence, the value of $(\log_a \frac{p}{q})^2$ is 1.

5) Answer: B

Explanation: To express 9 $\log_2 x$ + 3 $\log_2 y$ as a single logarithm, we need to apply the Properties of the Logarithm.

9 $\log_2 x$ and 3 $\log_2 y$ are the product of a quantity, and the logarithm of another quantity. The **Power Property of Logarithms** allows us to express this in the logarithm of the quantity raised to the power of the constant.

Hence, 9 $\log_2 x$ + 3 $\log_2 y$ as $\log_2 x^9 + \log_2 y^3$.

Now, we can simplify $\log_2 x^9 + \log_2 y^3$ by applying the **Product Property**:

$\log_2 x^9 + \log_2 y^3 = \log_2 x^9 y^3$.

To summarize:

9 $\log_2 x$ + 3 $\log_2 y$	
$\log_2 x^9 + \log_2 y^3$	*Power Property of Logarithms*
$\log_2 x^9 y^3$	*Product Property of Logarithms*