Sequence and Series

Answer Key

1) Answer: B

Solution: To find the 31st term of the sequence defined by the rule $a_n = \frac{1}{n+1}$, we simply substitute n = 31 to the given rule:

$a_n = \frac{1}{n+1}$

$a_{(31)} = \frac{1}{(31)+1}$ *Substitute n = 31*

$a_{31} = \frac{1}{32}$

Hence, the 31st term of the sequence defined by the rule $a_n = \frac{1}{n+1}$ is $\frac{1}{32}$.

2) Answer: D

Solution: To find the 72nd term of the given arithmetic sequence, we will use the formula for the nth term of an arithmetic sequence.

$a_n = a_1 + (n - 1)d$ *Formula for nth term of an arithmetic sequence*

The given arithmetic sequence is -9, -4, 1, 6, 11, … The first term of this sequence is -9, so we have a_1 = -9. Meanwhile, the common difference is 5 (-4 - (-9) = 5), so we have d = 5.

Since we are looking for the 72nd term, we will use n = 72.

Therefore, we have a_1 = -9, d = 5, and n = 72.

$$a_n = a_1 + (n - 1)d$$
$$a_{72} = (-9) + (72 - 1)5$$
$$a_{72} = (-9) + (71)5$$
$$a_{72} = (-9) + 355$$
$$a_{72} = 346$$

Therefore, the 72nd term of the given arithmetic sequence is 346.

3) Answer: D
Explanation: The arithmetic sequence in item 2 is -9, -4, 1, 6, 11, … If we are going to identify the sum of the first 100 terms of this sequence, then we'll have: (-9) + (-4) + 1 + 6 + 11 + …

Since we are adding terms of an arithmetic sequence, then (-9) + (-4) + 1 + 6 + 11 + … is actually an arithmetic series.

To find the sum of an arithmetic series, we use the formula:

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

- We have figured out from item 2 that the first term of the sequence is -9. Thus, we have $a_1 = -9$.
- The common difference is 5, so we have d = 5
- Since we want to identify the sum of its first 100 terms, then we have n = 100.

By substitution:

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$
$$S_{100} = \frac{100}{2}[2(-9) + (100 - 1)5]$$
$$S_{100} = 50[-18 + (99)5]$$
$$S_{100} = 50[-18 + 495]$$
$$S_{100} = 50[477]$$
$$S_{100} = 23850$$

Therefore, the sum of the first 100 terms of the given arithmetic sequence from the previous problem is 23,850.

4) Answer: A

Explanation: To find the nth term of a geometric sequence, we use the formula:

$$a_n = a_1 r^{n-1}$$

The given geometric sequence is -5, 5/2, -5/4, … Let us identify its first term (a_1), common ratio (r), and n.

- The first term of the geometric sequence is -5. Hence, a_1 = -5.
- The common ratio is $\frac{5}{2} \div -5 = \frac{5}{2} \div -\frac{5}{1} = \frac{5}{2} \times -\frac{1}{5} = -\frac{5}{10} = -\frac{1}{2}$. Thus, r = -½
- Since we are looking for the 11th term of the given sequence, then n = 11.

Substituting the values we have derived from our analysis above in the formula:

$$a_n = a_1 r^{n-1}$$
$$a_{11} = (-5)(\frac{-1}{2})^{10}$$
$$a_{11} = (-5)(\frac{1}{1024})$$
$$a_{11} = \frac{-5}{1024}$$

Thus, the 11th term of the geometric sequence is -5/1024.

5) Answer: B

Explanation: The given mathematical sentence which is $\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \ldots$ is an infinite geometric series. Note that this series has an "exact" sum since the absolute value of its common ratio (r) is ½ ($\frac{1}{6} \div \frac{1}{3} = \frac{1}{6} \times \frac{3}{1} = \frac{3}{6} = \frac{1}{2}$) which falls between 0 and 1.

The formula for the sum of an infinite geometric series is: $S_\infty = \frac{a_1}{1-r}$

- The first term of the sequence is $\frac{1}{3}$. Hence, we have $a_1 = \frac{1}{3}$
- The common ratio (r) of the sequence is $\frac{1}{2}$ (we have computed this earlier). So, $r = \frac{1}{2}$.

Substituting the derived values of a_1 and r to the formula:

$$S_\infty = \frac{a_1}{1-r}$$

$$S_\infty = \frac{\frac{1}{3}}{1-\frac{1}{2}}$$

$$S_\infty = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{1}{3} \div \frac{1}{2} = \frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$$

Hence, the sum of the given infinite geometric series is $\frac{2}{3}$.