Calculus has two major branches - differential calculus and integral calculus.

Differential calculus focuses on the concept of derivatives, how to derive them, and their application. We have already covered the basics of differential calculus in the previous reviewer.

This time, let's jump into the second branch of calculus, which deals with integrals. Just like derivatives, integrals offer a lot of practical applications in various fields.

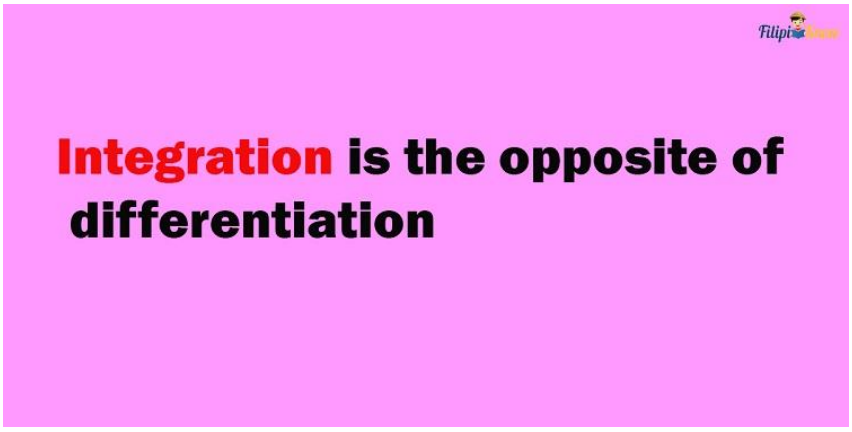
But what are integrals? How are they different from derivatives? How can you calculate them?

Let us answer these questions through this reviewer.

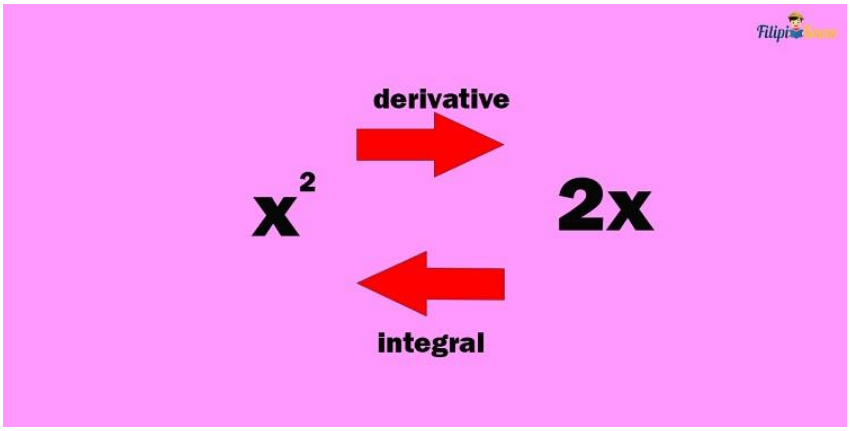
What Are Integrals?

Integrals are the "antiderivative" of a function. This means that the **integral is the opposite of the derivative**. The process of identifying the integral is known as **integration**.

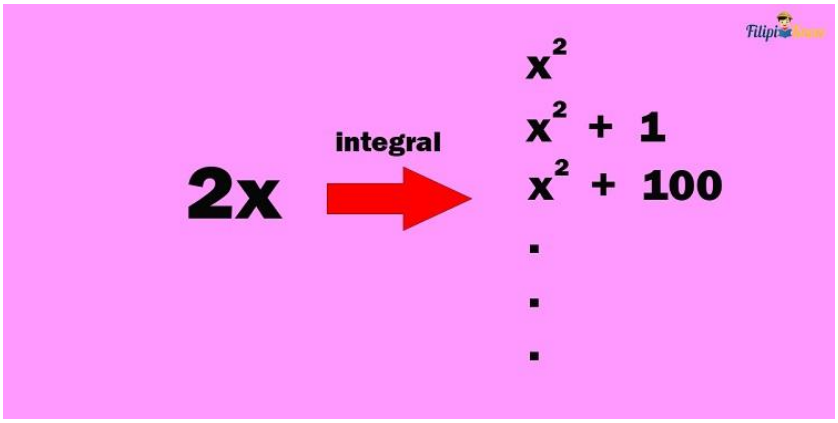
In integration, the given is the derivative of the function, and your goal is to compute the original function. That "original function" is the integral of the given function.



For example, suppose that we want to find the integral of 2x. This means that we need to find a function such that its derivative is equal to 2x. x^2 is an integral of 2x since the derivative of x^2 is equal to 2x:



Different functions can be an integral of 2x. For example, x^2, $x^2 + 1$, and $x^2 + 100$ are all possible integrals of 2x.

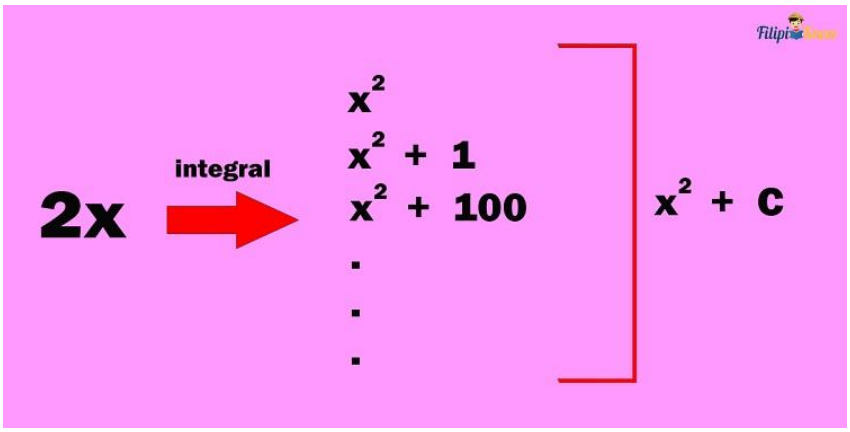


Because a given function takes multiple integrals, we add a letter “C” every time we have solved the integral. “C” stands for any arbitrary constant. This is why “C” is called the **constant of**

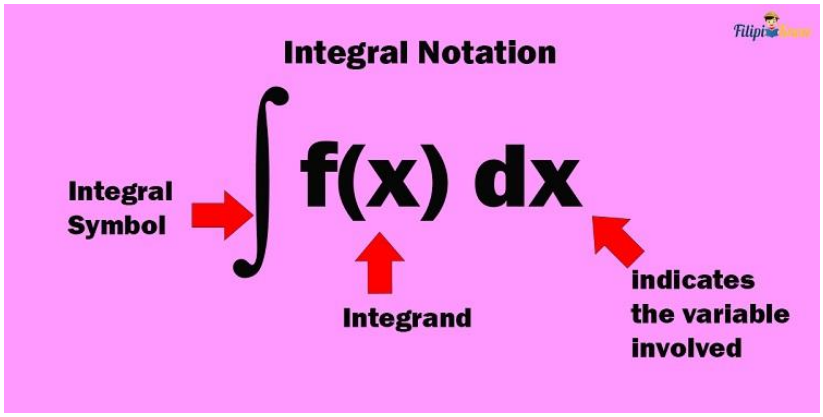
integration (actually, you can use any letter to represent the constant of integration, but for the sake of convenience, we will use “C” throughout this reviewer to refer to the constant of integration).

Going back to 2x, this function can have multiple integrals. For this reason, we provide a general solution that explains the integral of 2x. Specifically, we express the integral as $x^2 + C$, where C is an arbitrary constant. This means that C can take any numerical value such as 1, 1000, ½, π, 0.12, and so on.

Since $x^2 + 1$, $x^2 + 1000$, $x^2 + ½$, and $x^2 + π$ all have a derivative equal to 2x, we just generalize the integral as $x^2 + C$.



Integral Notation



When you see the integral symbol before a given function, it means that we are taking the integral of that function.

The function that we are taking the integral of is called the **integrand**. Note that there's always "*dx*" written after the given function to indicate the variable involved in the integration process (i.e., x).

For example, the notation $\int 3x^2\,dx$ means that we are integrating the function $3x^2$.

We need to identify a specific function whose derivative is equal to $3x^2$. In this case, the integrand is $3x^2$.

Indefinite and Definite Integrals

When taking the integral of a specific function, we must identify whether we are taking its indefinite integral or its definite integral.

The example above, $\int 3x^2\,dx$, is an example of a case where we are taking an indefinite integral.

In an indefinite integral, there are no upper limits and lower limits involved, and the answer is always a function with a variable and the constant of integration “C.”

On the other hand, definite integrals are those with upper and lower limits. Let’s take the integral below as an example.

$$\int_{3}^{4} 3x^2\,dx$$

The integral above have small numbers written above and below the integral sign. This is what we refer to as the limits of the integration. It means we are taking a definite integral. In a definite integral, the answer is not a function with a variable *x* but a whole number.

In this reviewer, we will primarily focus on indefinite integrals. However, we will also provide an overview of definite integrals to prepare you for an actual calculus class.

How To Find the Indefinite Integral of a Function

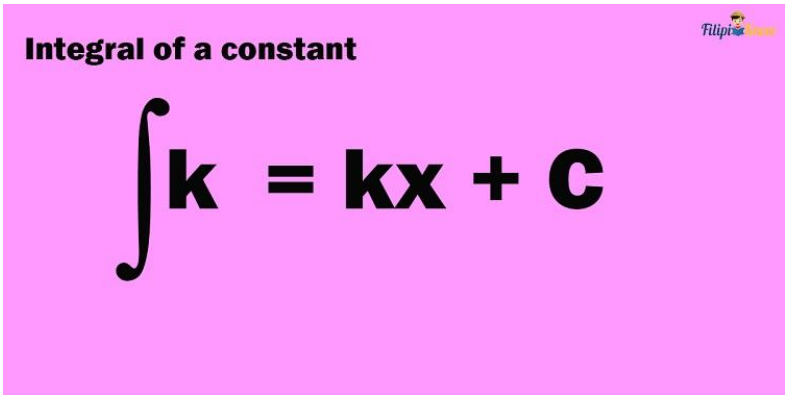
To find the indefinite integral of a function, we apply various integration rules. Note that the indefinite integral of a function is also a function with a variable and contains a constant of integration “C.”

Integration Rules

1. Integral of a Constant

“*The indefinite integral of a constant k is kx + C, where x is the variable involved.”*

In symbols:



This is the simplest of all the integration rules. It implies that the indefinite integral of a constant (or any numerical value) is just the constant itself times the variable involved in the integration plus the constant of integration C.

Sample Problem 1: Solve for $\int 2 \ dx$

Solution: To find the integral of 2, we multiply 2 by the variable involved (which is *x*) and then add C.

Hence, the indefinite integral of 2 is 2x + C.

Note that the derivative of 2x + C (where C is any numerical value), is equal to 2. Indeed, 2x + C is the integral of 2.

Sample Problem 2: Compute for $\int 9 \ dx$

Solution: Using the integral of a constant rule, we multiply 9 by *x* and then add C. Hence, the answer is 9x + C.

Sample Problem 3: What is $\int \pi \ dx$?

Solution: Note that the irrational number is also a constant (since it is still a numerical value). Hence, to find $\int \pi \ dx$, we need to apply the integral of a constant rule.

To solve for the indefinite integral, we just multiply by *x* and then add C. Hence, we have x + C as the answer.

Sample Problem 4: What is ∫ 9 dm?

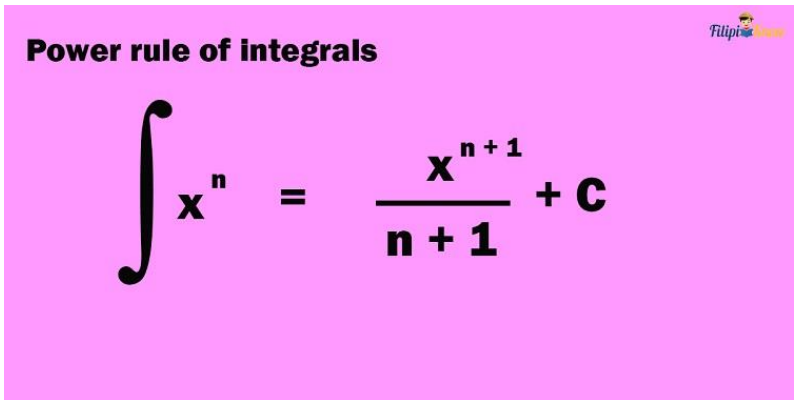
Solution: Note that the involved variable this time is not *x* but *m* instead. Hence, we should multiply 9 by *m* and then add C.

The answer is simply 9m + C.

2. Power Rule for Integrals

"The indefinite integral of x^n, where n is a real number, is equal to $\frac{x^{n+1}}{n+1} + C$."

The power rule allows us to identify the integral of a given function with a variable that is raised to a real number value. We can follow the formula below to find the integral of x^n (where *x* is variable and *n* is a real number).

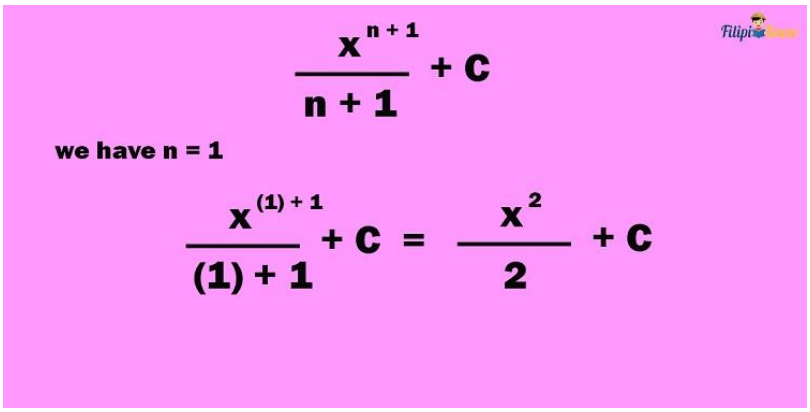


Thus, to find the integral of x^n, all you have to do is to identify first the value of *n* and then plug it into the formula above.

Sample Problem 1: Determine ∫ x dx

Solution: The given variable here is *x* with an exponent of 1 (note that *x* can be written as x^1). This means that we have *n* = *1*.

Let us plug n = 1 into the formula for the power rule.



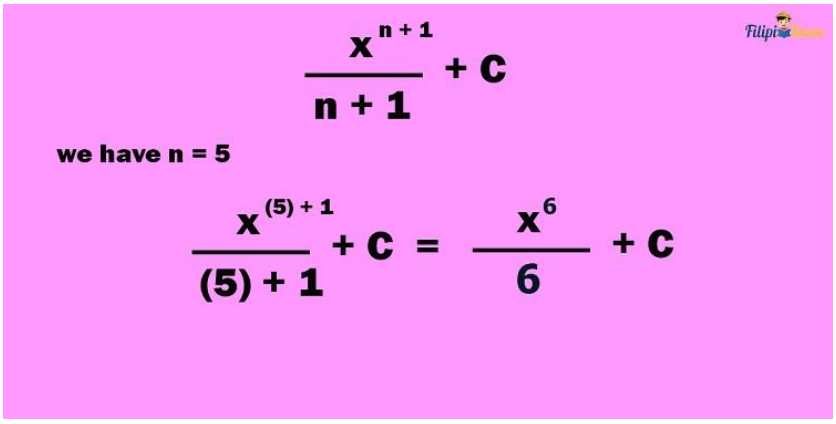
Hence, the indefinite integral of *x* is:

$$\frac{x^2}{2} + C \text{ or } \frac{1}{2}x^2 + C$$

Sample Problem 2: Compute for ∫ x^5 dx

Solution: The given variable here is *x* with an exponent of 5. This means that we have *n* = *5*.

Let us plug n = 5 into the formula for the power rule.



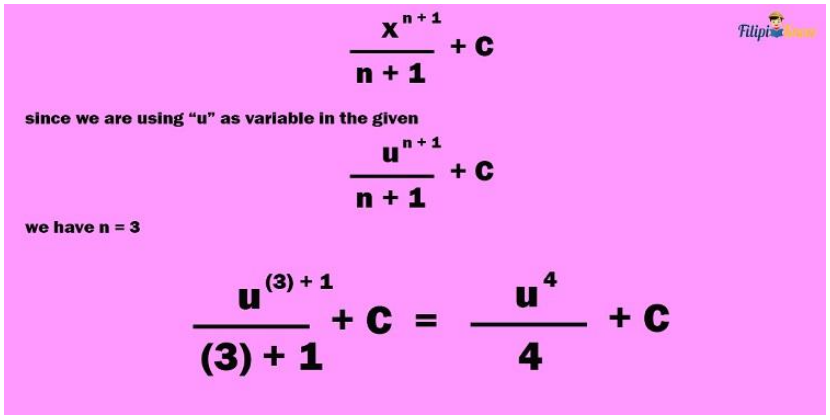
Hence, the indefinite integral of *x* is:

$$\frac{x^6}{6} + C \text{ or } \frac{1}{6}x^6 + C$$

Sample Problem 3: Compute for $\int u^3\, du$

Solution: The given variable here is *u* with an exponent of 3. This means that we have *n* = *3*.

Let us plug n = 3 into the formula for the power rule.



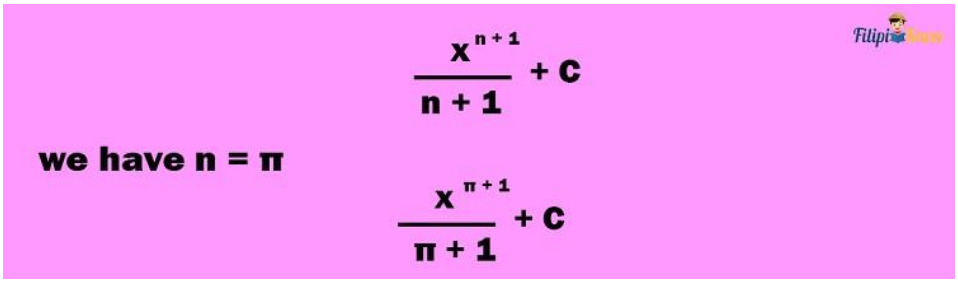
Hence, the indefinite integral of *u* is:

$$\frac{u^4}{4} + C \text{ or } \frac{1}{4}x^4 + C$$

Sample Problem 4: Use the power rule for integration to identify $\int x^{\pi}\, dx$

Solution: The given variable here is *x* with an exponent of π. This means that we have *n* = π.

Let us plug n = π into the formula for the power rule.



Hence, the indefinite integral of *u* is:

$$\frac{x^{\pi+1}}{\pi+1} + C \text{ or } \frac{1}{\pi+1}x^{\pi+1} + C$$

3. Multiplication of a Constant

"The indefinite integral of kx^n where k is a constant is equal to $k \int x^n dx$"

The multiplication of a constant integration rule tells us that the integral of the product of a constant and a function is equal to the product of the constant and the integral of the function.

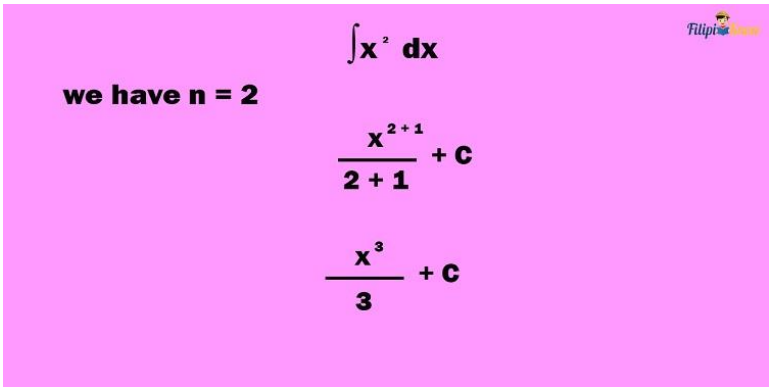
In symbols: $\int kx^n dx = k \int x^n dx$

Let's try to apply this rule to identify the integral of the product of a constant and a function.

Sample Problem 1: What is $\int 6x^2 dx$?

Solution: The integral of the product of a constant and a function can be expressed as the product of the constant and the integral of the function. This means that we can rewrite $\int 6x^2 dx$ as $6 * x^2 dx$.

The only thing we have to perform now is to compute for $\int x^2 dx$ using the power rule for integrals:



This means that

$$\int x^2 dx = \frac{x^3}{3} + C$$

Note that we have already expressed $\int 6x^2\,dx$ as $\int 6 * x^2\,dx$.

Therefore,

$$6 * \int x^2 dx = 6 * \frac{x^3}{3} + C$$

This means that

$$\int 6x^2\,dx = 6\frac{x^3}{3} + C$$

Note that we can write

$$6\frac{x^3}{3} + C \text{ as } \frac{6x^3}{3} + C$$

which is also equivalent to

$$\frac{6}{3}x^3 + C$$

6/3 in lowest terms is ½. Therefore:

The simplified form of

$$6\frac{x^3}{3} + C \text{ is } \frac{1}{2}x^3 + C$$

Here's a quick preview of what we have performed above:

$\int 6x^2\,dx$

$6\int x^2\,dx$ — *Multiplication of a constant rule*

$6 * \frac{x^3}{3} + C$ — *Power rule*

$\frac{6x^3}{3} + C$

$\frac{6}{3}x^3 + C$

$\frac{1}{2}x^3 + C$ — *Simplifying*

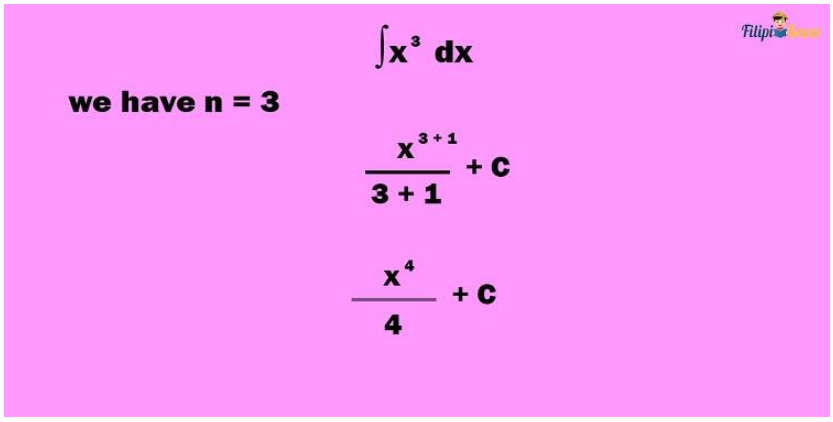
Sample Problem 2: Identify

$$\int \frac{1}{2}x^3\,dx$$

Solution: The integral of the product of a constant and a function can be expressed as the product of the constant and the integral of the function. This means that we can rewrite

$$\int \frac{1}{2}x^3\,dx \text{ as } \frac{1}{2} * \int x^3 dx$$

Let us apply the product rule for integrals so that we can identify the value of $\int x^3\,dx$:



This means that

$$\int x^3 dx = \frac{x^4}{4} + C$$

Since we already have ½ * ∫ x^3 dx earlier and we have computed that

$$\int x^3 dx = \frac{x^4}{4} + C$$

then we have

$$\frac{1}{2} * \frac{x^4}{4} + C$$

which can be simplified into

$$\frac{1}{8}x^4 + C$$

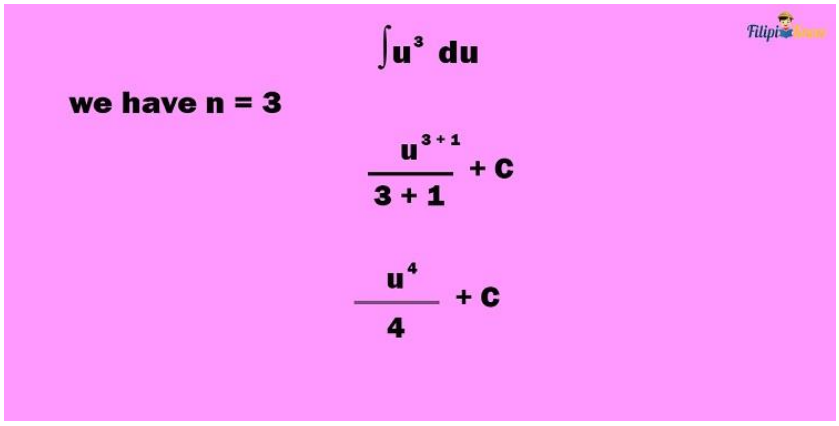
Hence, the answer for this problem is:

$$\frac{1}{8}x^4 + C$$

Sample Problem 3: Compute for $\int 0.10u^3\, du$

Solution: The integral of the product of a constant and a function can be expressed as the product of the constant and the integral of the function. This means that we can rewrite $\int 0.10u^3\, du$ as $0.10 * u^3\, du$.

Let us apply the product rule for integrals so that we can identify the value of $\int u^3\, du$:



This means that

$$\int u^3 du = \frac{u^4}{4} + C$$

Since we already have 0.10 * $\int u^3$ du earlier and we have computed that,

$$\int u^3 du = \frac{u^4}{4} + C$$

then we have

$$0.10 * \frac{u^4}{4} + C$$

which can be simplified into

$$\frac{0.10}{4}u^4 + C$$

Hence, the answer for this problem is:

$$\frac{0.10}{4}u^4 + C$$

4. Sum Rule of Integrals

"The indefinite integral of the sum of two functions is equal to the sum of the indefinite integral of the functions."

In symbols, ∫ [f(x) + g(x)] dx = ∫ f(x) dx + ∫ g(x) dx

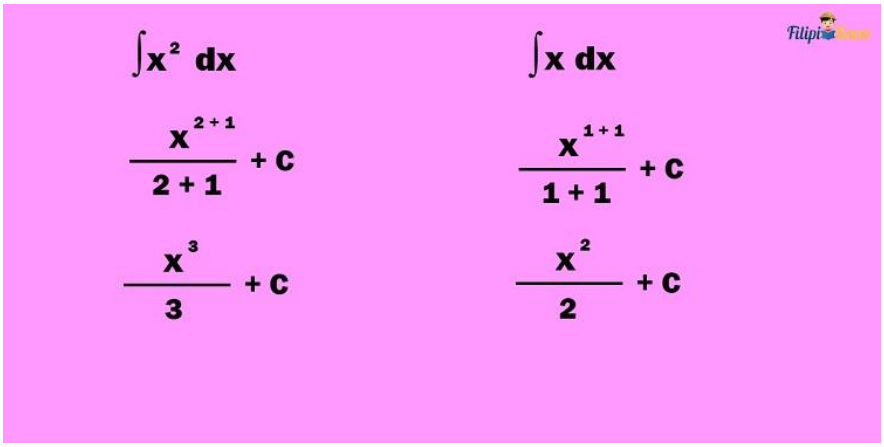
The sum rule allows us to find the integral of the sum of the functions by simply rewriting them as the sum of their respective integrals.

Sample Problem 1: Solve ∫ (x2 + x)dx using the sum rule.

Solution: In ∫ (x2 + x)dx, the integrand is the sum $x^2 + x$. This implies that we are taking the integral of the sum of two functions.

By applying the sum rule, we can express ∫ (x2 + x)dx as ∫ x2 dx + ∫ x dx.

Thus, our next move now is to determine the respective indefinite integrals of x^2 and x and then add them:



We combine the respective integrals of x^2 and x and consolidate the two constants of integration as a single constant (since the sum of two constants is also a constant).

Hence, the answer is

$$\frac{x^3}{3} + \frac{x^2}{2} + C$$

Here's a quick preview of what we have performed above:

$$\int (x^2 + x)\, dx$$

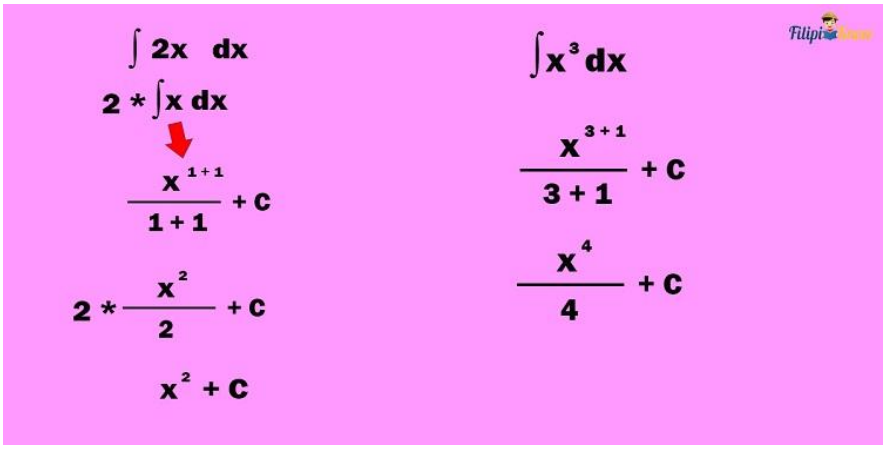
$$\int x^2\, dx + \int x\, dx \qquad \textit{Sum rule of integrals}$$

$$\frac{x^3}{3} + \frac{x^2}{2} + C \qquad \textit{Applying power rule for both integrals}$$

Sample Problem 2: Identify $\int (2x + x^3)\, dx$

Solution: By the sum rule of integrals, we can express $\int (2x + x^3)\, dx$ as $\int 2x\, dx + x^3\, dx$**.**

Now, let us derive the respective integrals of *2x* and x^3:



Hence, the answer is

$$x^2 + \frac{x^4}{4} + C$$

Here's a quick preview of what we have performed above:

$\int (2x + x^3)\, dx$

$\int 2x\, dx + \int x^3\, dx$ — *Sum rule of integrals*

$2\int x\, dx + \int x^3\, dx$ — *Multiplication of a constant rule*

$2*(\frac{x^2}{2}) + \frac{x^4}{4} + C$ — *Power rule*

$x^2 + \frac{x^4}{4} + C$ — *Simplify*

5. Difference Rule of Integrals

This is the counterpart of the sum rule of integrals for subtraction. Essentially, this rule states that *"the indefinite integral of the difference of two functions is equal to the difference of the integrals of the functions."*

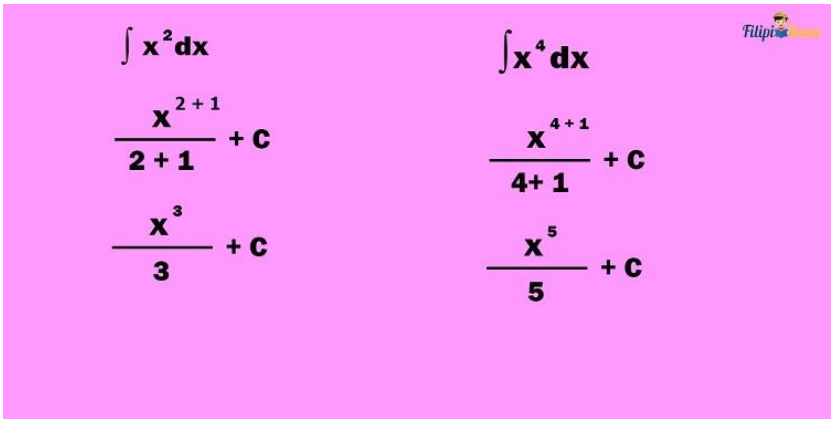
In symbols, $\int [f(x) - g(x)]\, dx = \int f(x)\, dx - \int g(x)\, dx$

Sample Problem 1: Identify $\int (x^2 - x^4)\, dx$

Solution: In the given expression above, the integrand is $x^2 - x^4$**,** which means we are integrating a difference of two functions. We can apply the difference rule for this case.

According to the difference rule, we can derive the integral of the difference between two functions by simply rewriting them as the difference of the integrals of the functions. Hence, we can rewrite $\int (x^2 - x^4)\, dx$ as $\int x^2\, dx - \int x^4\, dx$.

Let us now identify the indefinite integrals of x^2 and x^4 and subtract them so that we can solve this problem.



Using our solution above, we have identified that the integral is

$$\frac{x^3}{3} - \frac{x^5}{5} + C$$

Here's a quick preview of what we have performed above:

$$\int (x^2 - x^4)\, dx$$

$$\int x^2\, dx - \int x^4\, dx \qquad \textit{Difference rule of integrals}$$

$$\frac{x^3}{3} - \frac{x^5}{5} + C \qquad \textit{Power rule}$$

Sample Problem 2: Use the difference rule to identify $\int (3x^3 - ½ x^4) dx$

Solution: Using the difference rule, we can rewrite the given expression which is $\int (3x^3 - ½ x^4) dx$ as $3x^3 dx - \int ½ x^4 dx$.

Applying the integration rules we have learned, let us identify the respective integral of $3x^3$ and $½ x^4$.

Let us start with $3x^3$. We can express this as $3 * x^3$.

Using the multiplication of a constant rule, $\int (3 * x3)$ is equivalent to $3 * \int x3$

We can compute for the value of $\int x^3$ using the power rule for integrals:

$$\int x^3 = \frac{x^4}{4} + C$$

Hence,

$$\int (3 * x^3) = 3 * \frac{x^4}{4} + C$$

which is equivalent to

$$\frac{3x^4}{4} + C$$

Now, let us find the integral of ½ x^4.

Using the multiplication of a constant rule, ∫ (½ * x^4) = ½ ∫ x^4

We can compute the value of ∫ x^4 using the power rule for integrals:

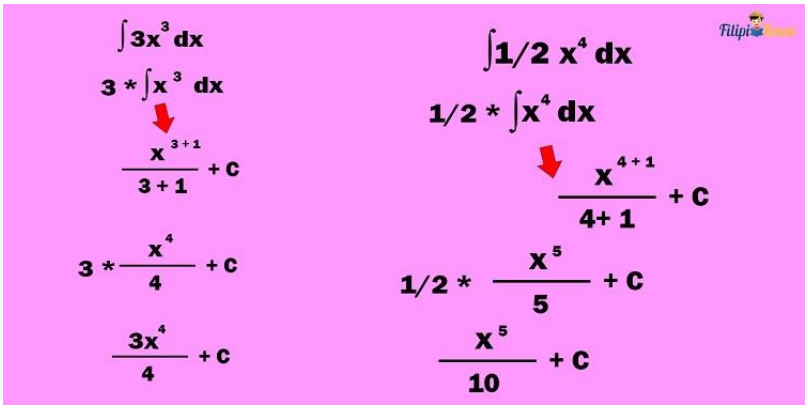
$$\int x^4 = \frac{x^5}{5} + C$$

Thus,

$$\int (\frac{1}{2} * x^4) = \frac{1}{2} * \frac{x^5}{5} + C$$

which is equivalent to

$$\frac{x^5}{10} + C$$



This means that the integral must be

$$\frac{3x^4}{4} - \frac{x^5}{10} + C$$

Sample Problem 3: Calculate for $\int (2x^3 + 3x^2 - x)dx$

Solution: Since the given integrand has addition and subtraction signs involved, we can apply the sum and difference rules simultaneously.

This implies that we can rewrite the given as $\int 2x^3\,dx + \int 3x^2\,dx - \int x\,dx$

Applying the multiplication by a constant rule, we have as follows:

$2 \int x^3\,dx + 3 \int x^2\,dx - \int x\,dx$

We then apply the power rule to calculate the respective integrals of x^3, x^2, and x:

$$2\left(\frac{x^4}{4} + C\right) + 3\left(\frac{x^3}{3} + C\right) - \left(\frac{x^2}{2} + C\right)$$

Rewriting the result above, we have:

$$\frac{x^4}{2} + x^3 - \frac{x^2}{2} + C$$

Hence, the integral is*:*

$$\frac{x^4}{2} + x^3 - \frac{x^2}{2} + C$$

Definite Integrals

In the previous section, we learned how to compute the indefinite integrals of basic functions. Remember that in an indefinite integral, the result is a function with a constant of integration “C’.

On the other hand, a definite integral of a function will give you a specific numerical value. Unlike indefinite integrals, definite integrals don't have to include the arbitrary constant "C" since we will obtain a specific value in this case.

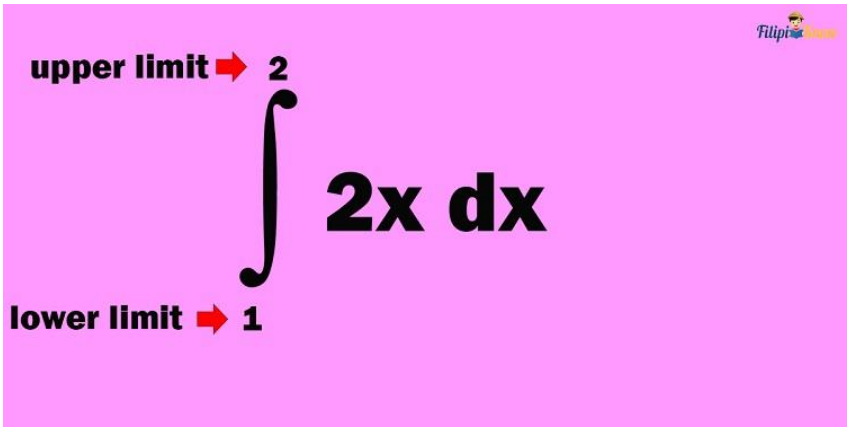
By including an upper limit and a lower limit to our integral symbol, we can evaluate the integral and obtain a numerical value.

For instance, the indefinite integral of $\int 2x\,dx = x^2 + C$. However, if we put an upper limit and lower limit on the given expression, we can obtain a numerical value.

Suppose that we have

$$\int_1^2 2x\,dx$$

If we solve this, we will obtain 3. Note that we get a numerical value, not a function with an arbitrary constant.



What is this “numerical value” we obtain in a definite integral? This is the area under the curve of the given integrand within the given limits. For instance, in

$$\int_{1}^{2} 2x\,dx$$

the definite integral of 3 means that the area under the function *2x* when graphed in the coordinate plane is equal to 3 within the range 1 and 2.

We will not delve too much into this analytical concept of integrals because it is already beyond the scope of our reviewer. For this reason, we will be focusing only on the algebra of calculating the definite integral.

How To Compute the Definite Integral

Suppose we want to find the definite integral given a lower limit *a* and an upper limit *b*. To find the definite integral:

1. Compute the indefinite integral of the given function.
2. Evaluate the indefinite integral at the lower limit *a*.
3. Evaluate the indefinite integral at the upper limit *b.*
4. Subtract the computed value in step 2 from the computed value in step 3.

Sample Problem: Find the definite integral of

$$\int_1^2 2x\,dx$$

Solution:

Step 1: *Compute the indefinite integral of the given function.*

By applying the integration rules we learned in the previous sections, we can compute the indefinite integral of the given function (2x).

$\int_1^2 2x\,dx$	
$2 * \int_1^2 x\,dx$	*Multiplication of a constant rule*
$2 * \frac{x^{1+1}}{1+1} + C$	*Power rule of integrals*
$2 * \frac{x^2}{2} + C$	
$x^2 + C$	*Simplify*

Step 2: *Evaluate the indefinite integral at the lower limit a.*

The lower limit in the given problem is 1. So, we substitute 1 in the computed indefinite integral in step 1, which is $x^2 + C$:

$x^2 + C$

$(1)^2 + C$ *Input the lower limit 1*

$1 + C$

Thus, we have obtained $1 + C$.

Step 3: *Evaluate the indefinite integral at the upper limit b.*

The upper limit in the given problem is 2. So, we substitute 2 in the computed indefinite integral in step 1, which is $x^2 + C$:

$x^2 + C$

$(2)^2 + C$ *Input the upper limit 2*

$4 + C$

Thus, we have $4 + C$.

Step 4: *Subtract the computed value in step 2 from the computed value in step 3.*

We obtained $4 + C$ in step 3 while $1 + C$ in step 2. Thus, we have:

$(4 + C) - (1 + C) = 3$

$(4 - 1) + (C - C) = 3$

Thus, the definite integral is 3.