In 1665, Isaac Newton invented calculus, a branch of mathematics that has changed the way we understand the world around us.

As a brilliant mathematician and physicist, he used calculus's rigors to explain some physics laws. From thereon, the applications of calculus have transcended physics and become useful in various fields such as chemistry, biology, engineering, statistics, economics, computer science, business analysis, and a lot more.

In this module, we'll begin our calculus study by learning the concept of limits.

An Introduction to Limits

Consider the function below

$$f(x) = \frac{x^2 - 2x + 1}{x - 1}$$

Recall that we can use any real number as the input value of *x* for this function and be assured that the output we will get is a real number, **except** when we substitute *x* = 1.

But why?

If we let *x* = *1* and input it in the function above, we will obtain a denominator of 0 which, as we know, is not defined in the set of real numbers. Recall that division by 0 is undefined.

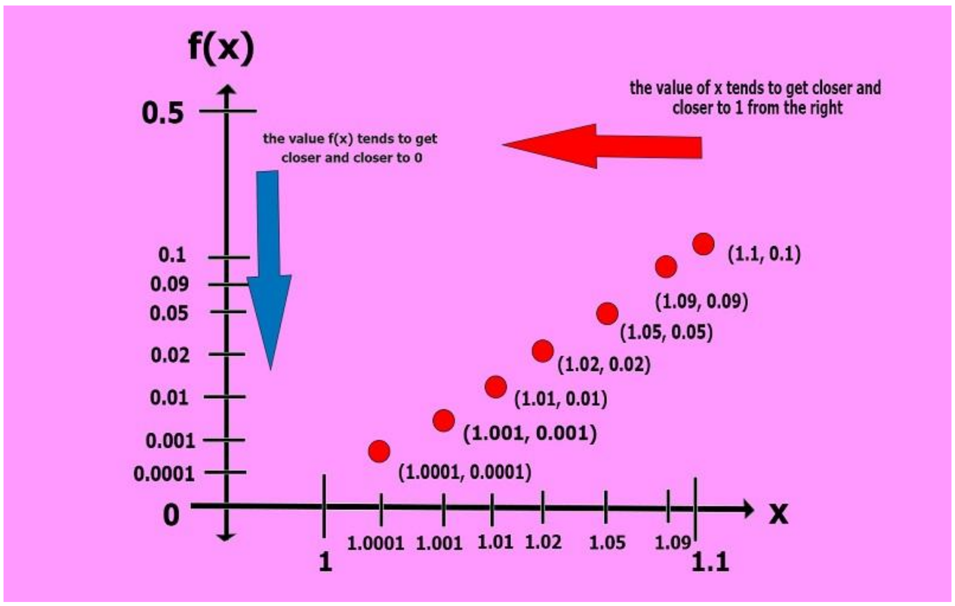
$$f(x) = \frac{x^2 - 2x + 1}{x - 1}$$

$$f(1) = \frac{(1)^2 - 2(1) + 1}{(1) - 1} = \frac{0}{0}$$

What will happen if we substitute values of *x* that are very near or approaching 1 instead? For instance, let us try to substitute the values $x_1 = 1.1$, $x_2 = 1.09$, $x_3 = 1.05$, $x_4 = 1.02$, $x_5 = 1.01$. Take note that these values of *x* are extremely near or we can say "approaching the value of *x* = 1". Also, note that these values are **on the right** of 1.

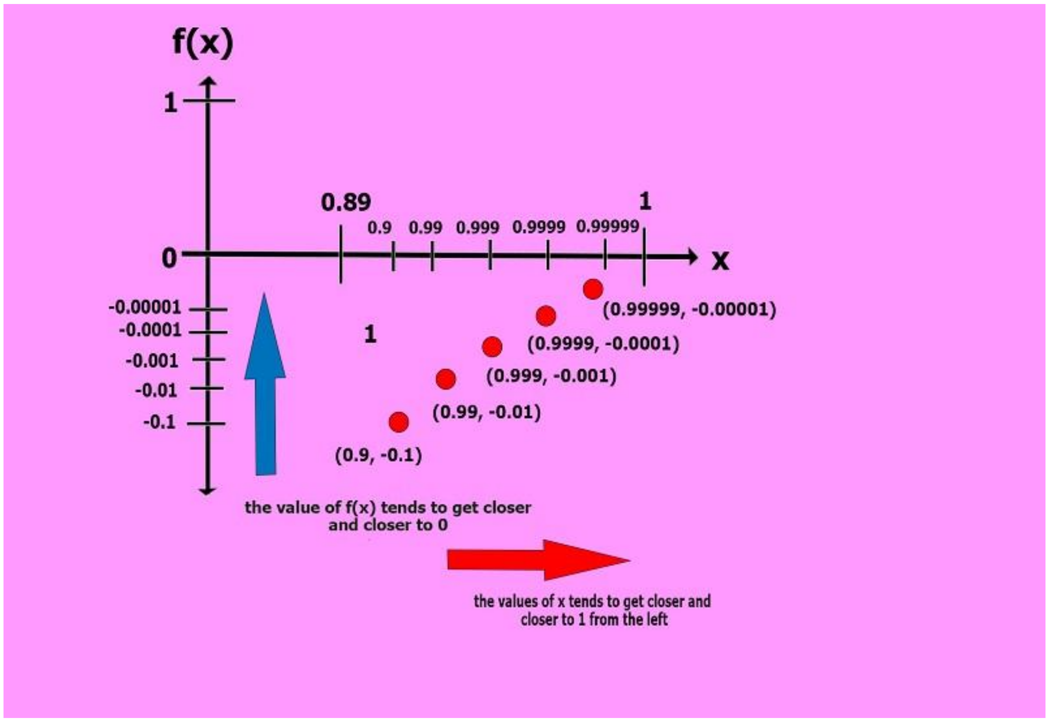
x	*f(x)*
1.1	0.1
1.09	0.09
1.05	0.05
1.02	0.02
1.01	0.01
1.001	0.001
1.0001	0.0001

Note that as the value of *x* gets closer and closer to the value of 1 from the right, the value of *f(x)* becomes closer and closer to 0. This means that if we insert the values of *x* that are extremely close to 1 from the right of it, the value of *f(x)* tends to approach 0.



In symbols, $x \to 1, f(x) \to 0$ ("as the value of *x* approaches 1, the value of *f(x)* approaches 0").

Now, let us consider values that are extremely close to 1 **from the left** such as $x_1 = 0.9$, $x_2 = 0.99$, $x_3 = 0.999$, $x_4 = 0.9999$, and $x_5 = 0.99999$. Again, these values are on the left of 1 and are extremely close to it.



x	*f(x)*
0.9	-0.1
0.99	-0.01
0.999	-0.001
0.9999	-0.0001
0.99999	-0.00001

As you can see, the closer the values of *x* to 1 from the left, the closer the value of *f(x)* to 0. We also see this behavior when we consider values close to 1 from the right. This means that in either direction (right or left), as the value of *x* becomes closer to 1, the value of *f(x)* tends to approach 0.

This means that the **limit** of the sample function above as *x* approaches 1 is 0.

Note that based on our analysis above, **substituting x = 1 to the function doesn't mean that the resulting value of *f(x)* is *0***; it only means that **the behavior of the function tends to reach 0 as we substitute values of *x* that are extremely close to 1 from the left of it or the right of it.**

From our example above, we can state the intuitive definition of limits.

Intuitive Definition of Limits

"Given that f(x) is a function and a is a real number, if the values of f(x) approach the real number value of L as the values of x approach or get closer and closer to a, then the limit of f(x) as x approaches a is the real number L."

Let's go back to our example:

$$f(x) = \frac{x^2 - 2x + 1}{x - 1}$$

The limit of the function is 0 as *x* approaches 1. This is because when substitute values of *x* get extremely close to 1, whether less than (on the left of) or greater than (on the right of) 1, the value of f(x) approaches 0.

It is important to note that even if the limit of f(x) is 0 as *x* approaches 1, it doesn't mean that f(1) = 0 or the value of f(x) is 0 when we let x = 1. Again, the value of f(x) would still be undefined at x = 1. The limit of 0 only tells us that the behavior of the function tends to reach 0 as we substitute values of *x* that are very close to 1.

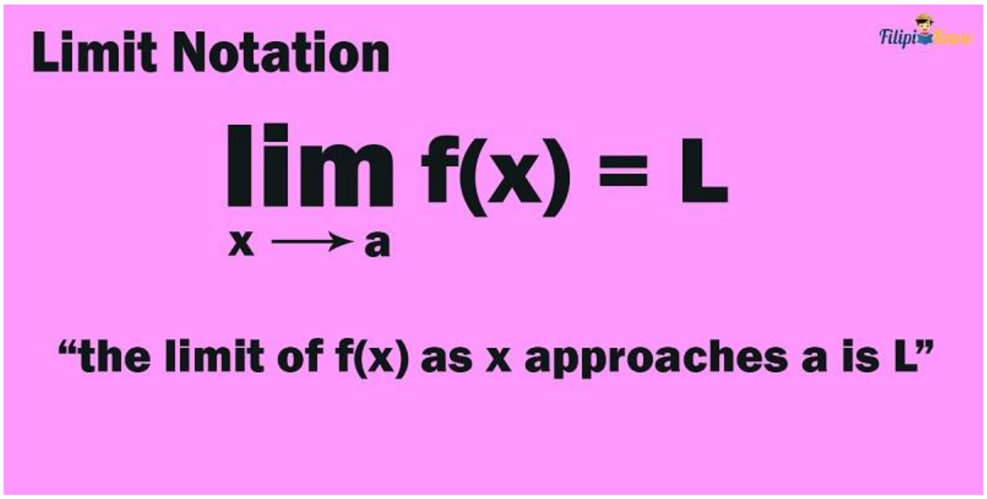
Again, note that the definition above is just an intuitive approach to defining what a limit is. The formal definition of it, known as the "Epsilon-delta definition for limits," is taught in higher mathematics classes so we will not cover it in this review.

Notation for Limits

To express the limit of a function concisely, we use a shorthand notation for it. If the limit of the function *f(x)* is a real number L as *x* approaches *a*, then we can write it as:

$$\lim_{x \to a} f(x) = L$$

The notation is read as "the limit of *f(x)* as *x* approaches *a* is *L*."



Going back to our earlier example,

$$f(x) = \frac{x^2 - 2x + 1}{x - 1}$$

If the limit of the function above is 0 as *x* approaches 1, then we can express it in this notation:

$$\lim_{x \to 1} \frac{x^2 - 2x + 1}{x - 1} = 0$$

Sample Problem: Express the following using the shorthand notation for limits

1. The limit of x + 3 as x approaches 3 is 6

2. The limit of

$$\frac{x^2 - 25}{x - 5}$$

as x approaches 5 is 10

Solution:

$$1.\ \lim_{x \to 3} x + 3 = 6$$

$$2.\ \lim_{x \to 5} \frac{x^2 - 25}{x - 5} = 10$$

Non-Existent Limits

Consider the function

$$f(x) = \frac{8}{x}$$

If we try to substitute *x* = 0 to this function, we will obtain an undefined value. So, let us consider values of *x* that are extremely close to 0 from the left and right of it.

Let's try to use the following values:

- **Values of *x* extremely close to 0 from the right:** x_1 = 0.2, x_2 = 0.1, x_3 = 0.05, x_4 = 0.01, x_5 = 0.0001, x_6 = 0.000001
- **Values of *x* extremely close to 0 from the left:** x_1 = -0.2, x_2 = -0.1, x_3 = -0.05, x_4 = -0.01, x_5 = -0.0001, x_6 = -0.000001

Now, let us evaluate the given function with the given values above.

Values of *x* extremely close to 0 from the right	f(x)	Values of *x* extremely close to 0 from the left	f(x)
0.2	40	-0.2	-40
0.1	80	-0.1	-80
0.05	160	-0.05	-160
0.01	800	-0.01	-800
0.0001	80 000	-0.0001	-80 000
0.000001	8 000 000	-0.000001	-8 000 000

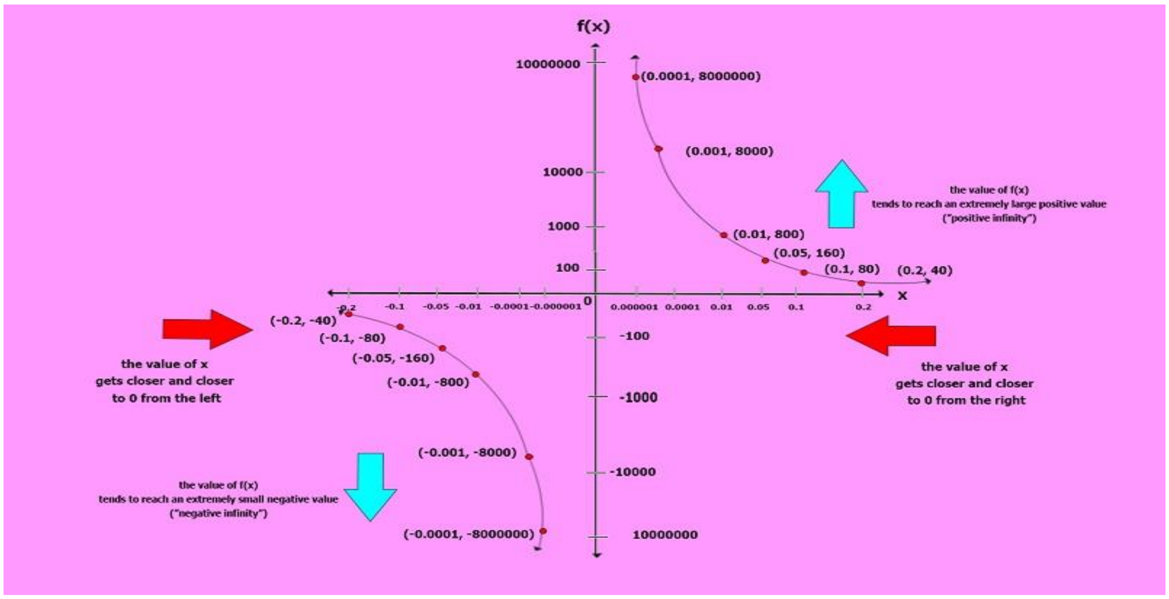
Notice that as *x* approaches 0 from the right, the value of *f(x)* becomes larger and larger without bounds. This means that **as *x* tends to reach 0 from the right, *f(x)* tends to reach an extremely large positive value or the so-called "positive infinity"** (Note: "positive infinity" is not a number or a quantitative value; it is just a term we use to state that a positive value is extremely large).

In symbols,

$$\lim_{x \to 0^+} \frac{8}{x} = +\infty$$

where the symbol "+" above 0 indicates values on the right of 0.

On the other hand, **as *x* approaches 0 from the left, the value of *f(x)* becomes smaller and smaller without bounds. This means that as *x* tends to reach 0 from the left, *f(x)* tends to reach an extremely small negative value or the so-called "negative infinity"** (Note: "negative infinity" is not a number or a quantitative value; it is just a term we use to state that a negative value is extremely small).



In symbols,

$$\lim_{x \to 0^-} \frac{8}{x} = -\infty$$

where the symbol "-" above 0 indicates values on the left of 0.

Whether the values of *x* that are extremely close to 0 are from the left of 0 or the right of 0, the limit of the function is not a certain number. Instead, **as *x* approaches 0 from the left or right, the value of *f(x)* = 8/x tends to become an extremely large or extremely small value.** Thus, we can conclude that the **limit of this function does not exist**.

In short, **if the functional value as *x* approaches a certain value is not a single number, then we can state that the limit is non-existent.**

There are various types of non-existent limits, but the most commonly encountered ones are **infinite limits**. Infinite limits imply that the functional values become extremely large or small as *x* approaches a certain value.

Our example above,

$$\lim_{x \to 0^+} \frac{8}{x} = +\infty$$

and

$$\lim_{x \to 0^-} \frac{8}{x} = -\infty$$

are examples of infinite limits.

Infinite limits do not tell us that the limits are infinite values per se; instead, it implies that no single whole number acts as a limit and the function values just tend to become larger and larger (or smaller and smaller) without bounds.

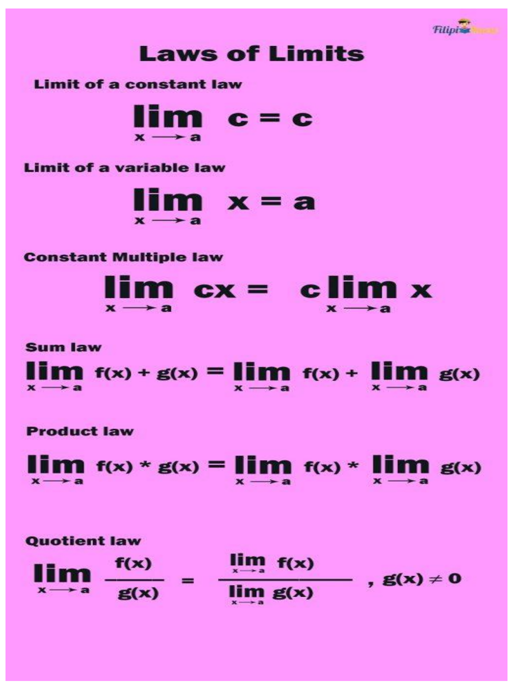
Laws of Limits

From the previous section, you learned how to identify the limits of a given function through the use of a table of values. However, this method is so tedious and will eat up a lot of our time.

For this reason, we have to use some mathematical devices to determine the limits of a function.

Fortunately, mathematicians have already done the hard work and provided us with the laws of limits—the mathematical tools we can use to determine limits. Let us discuss them one by one.

Overview of the Laws of Limits



1. Limit of a Constant Law

"The limit of a constant is the constant itself."

Limit of a constant law

$$\lim_{x \to a} c = c$$

This law tells us that whatever the value *x* approaches, the limit of a given constant is always the constant itself. In other words, just write the constant, and no need to do calculations.

Sample Problem 1: Evaluate

$$\lim_{x \to 2} 5$$

Solution: We are evaluating the limit of a constant (i.e., 5) in this example. By the limit of a constant law, the answer is just the constant itself. Thus, the answer is 5.

$$\lim_{x \to 2} 5 = 5$$

Sample Problem 2: Evaluate

$$\lim_{x \to 1000} \pi$$

Solution: Note that is a constant. Therefore, the answer for this example is just or

$$\lim_{x \to 1000} \pi = \pi$$

2. Limit of a Variable Law

"The limit of the variable *x* as *x* approaches *a* is just the value of *a.*"

Limit of a variable law

$$\lim_{x \to a} x = a$$

This law tells us that the limit of a certain variable *x* as *x* approaches *a* can be derived by simply replacing *x* with *a*.

Sample Problem: Evaluate

$$\lim_{x \to 1} x$$

Solution: By the limit of a variable law, we just replace *x* with 1, and the result is the limit. So,

$$\lim_{x \to 1} x = 1$$

3. Constant Multiple Law

"The limit of the product of a constant and a function is equal to the product of the constant and the limit of the function."

Constant Multiple Law

$$\lim_{x \to a} cx = c \lim_{x \to a} x$$

Let us have an example to understand this limit law:

Sample Problem 1: Evaluate

$$\lim_{x \to 1} 2x$$

Solution: Note that we can think of 2*x* as the product of *2* and the function or variable *x*. Hence, by the constant multiple law of limits:

$$\lim_{x \to 1} 2x = 2 \text{ x } \lim_{x \to 1} x$$

Recall that by the limit of a variable law,

$$\lim_{x \to 1} x = 1$$

Thus:

$$\lim_{x \to 1} 2x = 2 \text{ x } \lim_{x \to 1} x = 2 \text{ x } (1) = 2$$

The answer for this example is 2.

Sample Problem 2: Evaluate

$$\lim_{x \to 2} 0.5x$$

Solution: By the constant multiple law of limits:

$$\lim_{x \to 2} 0.5x = 0.5 \times (\lim_{x \to 2} x)$$

By applying the limit of a variable law,

$$\lim_{x \to 2} x = 2$$

Therefore:

$$\lim_{x \to 2} 0.5x = 0.5 \times (\lim_{x \to 2} x) = 0.5 \times 2 = 1$$

Thus, the answer is 1.

4. Sum Law

"The limit of a sum is equal to the sum of limits."

Sum Law

$$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

We can also apply the sum law on cases when subtraction is involved between limits of quantities.

Sample Problem 1: Evaluate

$$\lim_{x \to 1} x + 3$$

Solution: By the sum law of limits, we can express the limit of a sum as the sum of the respective limits of the addends.

$$\lim_{x \to 1} x + 3$$

$$\lim_{x \to 1} x + \lim_{x \to 1} 3$$ **Sum law**

Note that by the limit of a variable law, the

$$\lim_{x \to 1} x$$

can be derived by letting *x* = 1, so,

$$\lim_{x \to 1} x = 1$$

Meanwhile, by the limit of a constant law, the

$$\lim_{x \to 1} 3 = 3$$

Hence:

$$\lim_{x \to 1} x + \lim_{x \to 1} 3$$

$$1 + 3$$

$$= 4$$

Therefore,

$$\lim_{x \to 1} x + 3 = 4$$

Sample Problem 2: Evaluate

$$\lim_{x \to -1} x^2 + 3x - 2$$

Solution*:*

By applying the sum law of limits:

$$\lim_{x \to -1} x^2 + 3x - 2$$

$$\Downarrow$$

$$\lim_{x \to -1} x^2 + \lim_{x \to -1} 3x - \lim_{x \to -1} 2$$

By applying the limit of a variable law and limit of a constant law:

$$\lim_{x \to -1} x^2 + \lim_{x \to -1} 3x - \lim_{x \to -1} 2$$

$$(-1) + 3(-1) - 2$$

$$1 + 3(-1) - 2$$

$$1 - 3 - 2$$

$$-2 - 2$$

$$-4$$

Therefore,

$$\lim_{x \to -1} x^2 + 3x - 2 = -4$$

Sample Problem 3: Determine the limit of 2x2 - 3 as x approaches 0.5

Solution:

$$\lim_{x \to 0.5} 2x^2 - 3$$

By applying the sum law :

$$\lim_{x \to 0.5} 2x^2 - \lim_{x \to 0.5} 3$$

By applying the constant multiple law :

$$2\left(\lim_{x \to 0.5} x^2\right) - \lim_{x \to 0.5} 3$$

By applying the limit of a variable and limit of a constant law :

$$2\left((0.5)^2\right) - 3$$

$$2(0.25) - 3$$

$$0.5 - 3$$

$$= -2.5$$

Thus, the limit of $\boldsymbol{2x^2 - 3}$ as x approaches 0.5 is -2.5.

5. Product Law

"The limit of the product of functions is equal to the product of the limits of the functions."

$$\lim_{x \to a} \{f(x) \cdot g(x)\} = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

This law is quite intuitive and self-explanatory. We can rewrite the limit of a product as the product of the limits.

Sample Problem: Evaluate

$$\lim_{x \to 1} (x + 3)(x - 2)$$

Solution: By applying the product law, we can express the limit of the product of $x + 3$ and $x + 2$ as the product of the respective limits of $x + 3$ and $x + 2$.

$$\lim_{x \to 1} (x + 3) \cdot (x - 2)$$

$$\lim_{x \to 1} (x + 3) \cdot \lim_{x \to 1} (x - 2)$$

Let us continue evaluating the limit by applying the limit laws we have learned earlier:

$$\lim_{x \to 1} (x + 3) \cdot \lim_{x \to 1} (x - 2)$$

By sum law :

$$\left(\lim_{x \to 1} x + \lim_{x \to 1} 3\right) \bullet \left(\lim_{x \to 1} x + \lim_{x \to 1} 2\right)$$

$$((1) + 3) \bullet ((1) - 2)$$

$$4 \bullet (-1)$$

$$= -4$$

Thus, the answer to this problem is -4.

6. Quotient Law

"The limit of the quotient of functions is equal to the quotient of the limits of the functions."

$$\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$

$$\text{Where } \lim_{x \to a} g(x) \neq 0$$

Quotient law is almost similar to the product law, but instead of multiplication, we are now dealing with division. Through this law, we can rewrite the limit of a quotient as the quotient of the limits.

Sample Problem: Evaluate

$$\lim_{x \to 1} \frac{x+3}{x-2}$$

Solution: By applying the quotient law, we can express the limit of the quotient of x + 3 and x + 2 as the quotient of the respective limits of x + 3 and x + 2:

$$\lim_{x \to 1}\left\{\frac{x+3}{x-2}\right\} = \frac{\lim_{x \to 1} x+3}{\lim_{x \to 1} x-2}$$

Let us continue evaluating the limit by applying the limit laws we have learned earlier:

$$\frac{\lim_{x \to 1} x+3}{\lim_{x \to 1} x-2}$$

$$\frac{\lim_{x \to 1} x + \lim_{x \to 1} 3}{\lim_{x \to 1} x - \lim_{x \to 1} 2}$$

$$= \frac{1+3}{1-2}$$

$$= \frac{4}{-1}$$

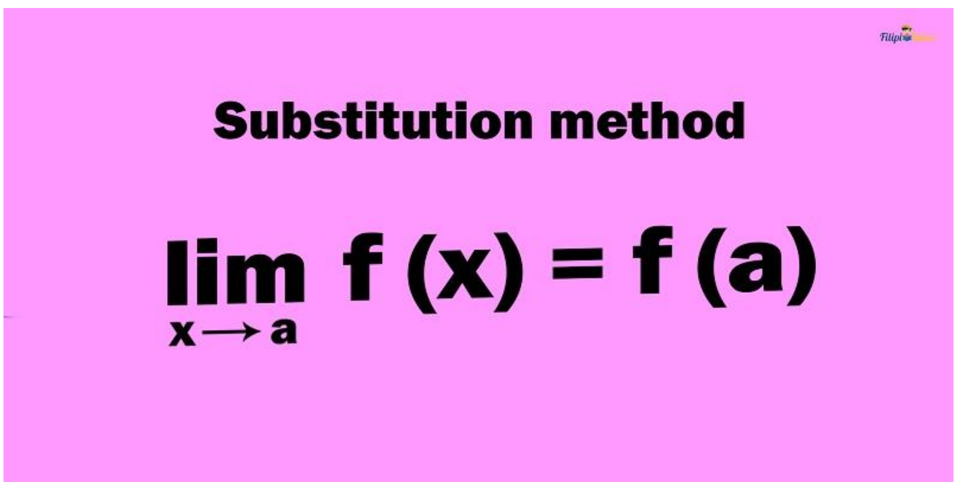
$$= -4$$

Thus, the limit is -4.

Evaluating the Limit Using the Substitution Method

In the previous section, you have learned how to evaluate the limit of a function by applying the limit laws. However, you might have realized that applying the laws is still quite tedious since you have to write the limit notation repeatedly.

In this section, you are going to learn how to evaluate the limit of a function through a substitution method, which allows you to just substitute *a* to *x* to find the limit of *f(x)* as *x* approaches *a*:



However, you can only apply the substitution method if the limiting value is an element of the domain of the function or if that value of *x* will not make the function undefined.

Let us apply the substitution method to find the limit of $x^2 - 2x + 1$ if *x* approaches 1.

If we use the limit laws for this case, it will be quite fast but also tedious since we have to write the limit notation repeatedly. The limiting value, 1, is part of the domain of $x^2 - 2x + 1$; it will not make the value of the function undefined if we substitute it. Therefore, we can use the substitution method to find the limit of the given function:

$$\lim_{x \to 1} x^2 - 2x + 1 = (1) - 2(1) + 1 = 1 - 2 + 1$$
$$= -1 + 1$$
$$= 0$$

Hence, the answer is 0.

Sample Problem 1: What is the limit of the function *f(x) = 2x - 5* as *x* approaches -2?

Solution: We can apply the substitution method for this case since the limiting value of -2 will not make the function undefined.

By substitution method:

$$2(-2) - 5$$

$$-4 - 5 = -9$$

Hence, the answer is -9.

Sample Problem 2: Determine the limit of the function $f(x) = 4x - x^2$ as x approaches ½ .

Solution: We can apply the substitution method for this case since the limiting value of ½ will not make the function undefined.

$$f(x) = 4x - x^2$$

Substituting x = ½:

$$4(½) - (½)^2$$

Note that $(½)^2$ = ½ x ½ = ¼. So, we have:

$$2 - ¼$$

To find the difference between 2 and ¼, we need to make the denominators similar.

$$8/4 - 1/4$$

$$= 7/4$$

Thus, the answer is 7/4.

However, there are some cases where the substitution method cannot be used directly to evaluate limits.

For instance, if we try to determine the limit of

$$\frac{x^2 - 16}{x - 4}$$

as x approaches 4, we cannot use the substitution method directly because if we substitute x = 4 to the function, we will have a 0 denominator and lead to a value that is not a real number. Hence, we cannot use the substitution method for this case.

In the succeeding section, you will learn how to evaluate the limits of functions when the substitution method is not applicable to them.

Evaluating Limits Using the Factoring Method

If we cannot apply the substitution method to identify the limit of a function, one method we can use is the factoring method.

Take a look again at the function below

$$f(x) = \frac{x^2 - 16}{x - 4}$$

Let's try to identify the limit of this function as *x* approaches 4. Suppose that we use the substitution method to find the limit.

$$\lim_{x \to 4} f(x) = \frac{x^2 - 16}{x - 4} = (4)^2 - 16 / 4 - 4 = \frac{0}{0}$$ *By substitution method*

Clearly, the substitution method fails to find the limit of the function. We have obtained 0/0 which is not a real number. 0/0 is one of the so-called **indeterminate forms** in calculus.

Limits in the indeterminate form 0/0 can be solved using the factoring method.

Sample Problem 1: Can we use the factoring method for these limits?

a) $\lim_{x \to -2} \frac{x^2 - 4}{x + 2}$

b) $\lim_{x \to -5} \frac{x^2 - 25}{x + 5}$

c) $\lim_{x \to 0} \frac{5}{x}$

Solution:

The limit in (a) can be solved by the factoring method since it is in the indeterminate form 0/0 if we substitute $x = -2$ to the function.

The limit in (b) can be solved by the factoring method since it is in the indeterminate form 0/0 if we substitute x = -5 to the function.

The limit in (c) cannot be solved by the factoring method since it will not result in an indeterminate form 0/0 if we substitute x = 0 to the function. If we substitute x = 0, we will obtain 5/0 instead, which is not the indeterminate form 0/0.

To evaluate limits in indeterminate form 0/0 using the factoring method, **we just factor the expression in the numerator or the denominator, cancel the common factor, then apply the substitution method.**

Sample Problem 2: Evaluate

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$$

Solution: The given limit is in the indeterminate form 0/0 if we substitute x = 4 to the function.

Note that we can factor x^2 - 16 as (x + 4)(x - 4) (factoring difference of two squares):

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4} = \lim_{x \to 4} \frac{(x + 4)(x - 4)}{x - 4}$$

We can cancel out the common factor x - 4:

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4} = \lim_{x \to 4} \frac{(x + 4)\cancel{(x - 4)}}{\cancel{x - 4}}$$

$$= \lim_{x \to 4} (x + 4)$$

Lastly, we apply the substitution method:

$$\lim_{x \to 4} (x + 4) = (4) + 4 = 8$$

Therefore,

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4} = 8$$

Sample Problem 3: Evaluate

$$\lim_{x \to 4} \frac{x^2 - 9x + 20}{x - 4}$$

Solution: The given limit is in indeterminate form 0/0 if we substitute x = 4 to the function.

Thus, we can solve the given limit by factoring:

$$\lim_{x \to 4} \frac{x^2 - 9x + 20}{x - 4}$$

$$\lim_{x \to 4} \frac{(x - 5)(x - 4)}{x - 4}$$

factoring the numerator

$$\lim_{x \to 4} \frac{(x - 5)\cancel{(x - 4)}}{\cancel{x - 4}}$$

Cancel out common factor

$$\lim_{x \to 4} x - 5$$

$$= 4 - 5$$

$$= -1$$

Thus, the limit is -1.

Sample Problem 4: Evaluate

$$\lim_{x \to -1} \frac{2x(x + 5) + 3}{x + 1}$$

Solution: By the distributive property, we can rewrite the given function as

$$\lim_{x \to -1} \frac{2x(x+5)+3}{x+1} = \lim_{x \to -1} \frac{2x^2+5x+3}{x+1}$$

Thus, we are just basically determining or evaluating

$$\lim_{x \to -1} \frac{2x^2+5x+3}{x+1}$$

If we substitute x = -1 to the function, we will obtain the indeterminate form 0/0. Thus, we can use the factoring method to solve for the limit.

$$\lim_{x \to -1} \frac{2x^2 + 5x + 3}{x + 1}$$

$$\lim_{x \to -1} \frac{(2x + 3)(x + 1)}{x + 1}$$ **by factoring**

$$\lim_{x \to -1} 2x + 3$$

$$= 2(-1) + 3$$ **By substitution method**

$$= -2 + 3$$

$$= 1$$

The answer to this problem is 1.

Infinite Limits

In the earlier section, we discussed what infinite limits are. Essentially, this happens when we get the limit of a function as x approaches a particular value but it never gets closer to a real number. What's happening is that the value of f(x) tends to get larger and larger or smaller and smaller.

An infinite limit means the limit also does not exist.

Sample Problem: Consider

$$f(x) = \frac{3}{x}$$

What is the limit of f(x) as *x* approaches 0?

Solution: We cannot use the substitution method or the factoring method for this case. First, 0 will make the function undefined, and it will also not lead to an indeterminate form 0/0 in case we substitute it (3/0 instead).

So, how do we identify its limit without using an exhaustive table of values?

Here's our technique:

We substitute values of *x* close to 0, one from the right of *0* (greater than x) and one on the left of 0 (less than 0).

For the left of 0, let us use -0.001 (the symbol "-" above 0 in the limit below indicates that we are using a value of *x* on the left of 0).

$$\lim_{x \to 0^-} \frac{3}{x} \qquad \left(\frac{3}{-0.001} = -3000\right)$$

Notice that the value we have obtained is a small negative number. If we try to substitute a value that is much closer to 0, we will surely obtain a much smaller negative number. Then, we can state that as *x* approaches 0 from the left, the value of *f(x)* becomes smaller and smaller without bounds.

Meanwhile, for the right of 0, if we use 0.001 (the symbol "+" above 0 in the limit below indicates that we are using a value of x on the right of 0).

$$\lim_{x \to 0^+} \frac{3}{x} \qquad \left(\frac{3}{0.001} = 3000\right)$$

Notice that the value we have obtained is a large positive number. If we try to substitute a value that is much closer to 0, we will surely obtain a much larger positive number. Then, we can state that as *x* approaches 0 from the right, the value of *f(x)* becomes larger and larger without bounds.

We can summarize our observation below:

$$\lim_{x \to 0^-} \frac{c}{x} = -\infty \qquad \lim_{x \to 0^+} \frac{c}{x} = +\infty$$

Where c is a positive real number.

In our next chapter, you are going to learn differentiation - the heart of calculus. The concept of the limits of functions is the building block to understanding differentiation. Hence, we advise you to keep reading this reviewer to absorb the concepts and prepare for the next chapter.