Propositional Logic

Answer Key

1) Answer: A

Explanation: Let us identify the respective truth value of each statement:

(a) True, all negative numbers have a negative cube. Let *-n* be a negative real number. If we multiply *-n* by itself thrice: $-n \times -n = n^2 \times -n = -n^3$ the result is a negative number.
(b) False, ½ is a rational number between 0 and 1.
(c) False, other numbers are multiples of 5 and located between 10 and 50. For instance, 25 is a multiple of 5 between 10 and 50.
(d) False; 2 is the smallest prime nonnegative number.

Based on our analysis above, the statement with a truth value of true is option A.

2) Answer: A

Explanation: $\sim r \vee p \Rightarrow q$

Let *p* be the proposition *"There is no rational number between 0 and 1,"* *q* be the proposition "*2 is the smallest prime nonnegative integer"* and *r* be the proposition *"$3^2 = 9$."*

∼r indicates the negation of proposition *r* which is "$3^2 = 9$." Proposition *r* has a truth value of true which implies that its negation, ∼r, must have a false truth value.

$\sim r \vee p$ is the disjunction of the negation of *r* and proposition *p*. Proposition *p* states a false proposition since there exists a rational number between 0 and 1 (e.g., ½, 0.10, etc.). Meanwhile, recall that we found out earlier that $\sim r$ is false. Since both of the propositions $\sim r$ and *p* are false, then the disjunction $\sim r \vee p$ must be false.

To identify the truth value of the entire statement, $\sim r \vee p \Rightarrow q$, we must identify first the hypothesis and the conclusion of this conditional statement. Clearly, $\sim r \vee p$ is the hypothesis while *q* is the conclusion. Since we stated earlier that $\sim r \vee p$ is false, it means that $\sim r \vee p \Rightarrow q$ is true regardless of the truth value of *q* (recall that in a conditional statement, once the hypothesis is false, then the statement is already true regardless of the truth value of the conclusion).

Hence, the answer to this problem is false.

3) Answer: A

Explanation: Based on item 2, we have the following propositions and their respective propositional variables:

- Proposition *p: "There is no rational number between 0 and 1"*
- Proposition *q:* "*2 is the smallest prime nonnegative integer"*
- Proposition *r*: "$3^2 = 9$"

Our goal is to translate *"If* $3^2 = 9$*, then 2 is not the smallest prime nonnegative integer or there exists a rational number between 0 and 1"* into a statement with propositional variables *p, q,* and *r*.

Since we are dealing with an "If and then" statement, then we expect that we are going to use the implication symbol (⇒). We have to identify what we need to place on the left side of (the hypothesis) and the right side (the conclusion) of the arrow.

The hypothesis of the conditional statement is "$3^2 = 9$" represented by the proposition *r*. Therefore, we need to put *r* on the left side of the implication arrow.

Meanwhile, the conclusion of the conditional statement is "*2 is not the smallest prime nonnegative integer or there exists a rational number between 0 and 1."* This statement is a disjunction of the negation of proposition *q* and the negation of proposition *p*. Hence, our conclusion is symbolized by $\sim q \vee \sim p$. This implies that we need to put $\sim q \vee \sim p$ on the right side of the arrow.

Our final answer will be $r \Rightarrow \sim q \vee \sim p$

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4) Answer: D

Explanation: To identify the correct answer, we take the converse of each statement. Recall that the converse of a conditional statement is obtained by simply interchanging the hypothesis and the conclusion of the statement.

Given statements:

I. If a number is an integer, then it is also a rational number.
II. If an organism is a man, then that organism can run.
III. If an angle is obtuse, then the angle is not an acute angle.
IV. If a segment is the diameter of a circle, then that segment is also a chord.

Converse:
I. If a number is rational, then it is an integer (False, since there are rational numbers that are not integers, such as ½, 0.5, 0.1, etc.).
II. If an organism can run, then the organism is a man (False; some organisms can run, and they are nonhumans).
III. If an angle is not acute, then the angle is obtuse (False; if an angle is not acute, it doesn't necessarily mean that the angle is an obtuse angle. It can either be an obtuse angle, right angle, or a straight angle).
IV. If a segment is a chord, then that segment is the diameter of a circle (False; not all chords are the diameter of a circle).

Hence, all given statements are false.

5) Answer: C

Explanation: To form the contrapositive of a conditional statement, all we have to do is negate its hypothesis and conclusion. Make the negated hypothesis the new conclusion, and then the negated conclusion the new hypothesis.

Propositional Logic

Answer Key

The given statement is "*If x + y is divisible by 3, then xy is not divisible by 3".*

The hypothesis of this statement is "*x + y is divisible by 3"*. The negation of this statement is "*If x + y is not divisible by 3"*. Meanwhile, the conclusion is *"xy is not divisible by 3"*. The negation of this statement is *"xy is divisible by 3"*.

Therefore, the contrapositive of the statement should be *"If xy is divisible by 3, then x + y is not divisible by 3*".