Truth Tables and Logical Equivalence

Answer Key

1) Answer: B

Explanation: We can easily identify the truth value of the given statement by creating a truth table.

We prepare a table with 7 columns since we have three propositions (*p, q,* and *r*) and four connectives involved (2 negations, 1 implication, and 1 disjunction).

p	q	r	$\sim p$	$q \vee r$	$\sim (q \vee r)$	$\sim p \Rightarrow \sim (q \vee r)$

The problem states that *p* and *r* are both true while *q* is false.

p	q	r	$\sim p$	$q \vee r$	$\sim (q \vee r)$	$\sim p \Rightarrow \sim (q \vee r)$
T	F	T				

We can easily identify that the negation of *p* must be F.

p	q	r	$\sim p$	$q \vee r$	$\sim (q \vee r)$	$\sim p \Rightarrow \sim (q \vee r)$
T	F	T	F			

*q v r (*a disjunction) must be T since one of *q* and *r* is true.

p	q	r	$\sim p$	$q \vee r$	$\sim (q \vee r)$	$\sim p \Rightarrow \sim (q \vee r)$
T	F	T	F	T		

To God be the glory!

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The expression $\sim (q \vee r)$ means the negation of $q \vee r$. Since we have discovered earlier that $q \vee r$ is true, then its negation must be false.

p	q	r	$\sim p$	$q \vee r$	$\sim (q \vee r)$	$\sim p \Rightarrow \sim (q \vee r)$
T	F	T	F	T	F	

Lastly, we refer to columns 5 and 6 to identify the truth value of $\sim p \Rightarrow \sim (q \vee r)$. Note that we have a true hypothesis and a false conclusion. Hence, $\sim p \Rightarrow \sim (q \vee r)$ must be false.

p	q	r	$\sim p$	$q \vee r$	$\sim (q \vee r)$	$\sim p \Rightarrow \sim (q \vee r)$
T	F	T	F	T	F	F

The answer is false.

2) Answer: C

Explanation: Recall that the contrapositive of a conditional statement is logically equivalent to it (Law of Contrapositive). Hence, the statement *"If Mina did not win the lottery, then she would not have bought a new car,"* which is the contrapositive of the statement, is the one that is logically equivalent to the given statement.

3) Answer: A

Explanation: We create truth tables for each statement and determine which is the tautology:

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For option A:

p	q	$p \wedge q$	$(p \wedge q) \Rightarrow p$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	F	T

Since we have already shown that option A is a tautology, then we are assured that A can be an answer, but not yet since there is an option D that states that both A and C are tautologies.

Let's try to identify whether C is a tautology or not:

For option C:

p	q	$p \wedge q$	$(p \wedge q) \Rightarrow q$
T	T	T	T
T	F	T	F
F	T	F	T
F	F	F	T

Clearly, the one in statement C is not a tautology.

Hence, we have already confirmed that the statement in A is the only statement that is a tautology among the given options.

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4) Answer: A

Explanation: We create a truth table to show the truth value of $\sim p \wedge q \Leftrightarrow p \vee \sim q$ given that *p* and *q* are both false.

There are two propositions involved (*p* and *q*) and five connectives involved (2 negations, 1 conjunction, 1 disjunction, and 1 biconditional). Hence, we prepare a table with 7 columns:

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$\sim p \wedge q$	$\sim p \wedge q \Leftrightarrow p \vee \sim q$

Since *p* and *q* are both false, then we have:

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$\sim p \wedge q$	$\sim p \wedge q \Leftrightarrow p \vee \sim q$
F	F					

The respective negations of *p* and *q* must be T:

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$\sim p \wedge q$	$\sim p \wedge q \Leftrightarrow p \vee \sim q$
F	F	T	T			

$\sim p \wedge q$ must be false since not both $\sim p$ and *q* are true. Same as with $\sim p \wedge q$.

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$\sim p \wedge q$	$\sim p \wedge q \Leftrightarrow p \vee \sim q$

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F	F	T	T	F	F	

In a biconditional statement, the expression on the left-hand side and the right-hand side of the arrow must have the same truth value so that its truth value is T. Referring to our table above, both sides of $\sim p \wedge q \Leftrightarrow p \vee \sim q$ have a truth value of F. Hence, $\sim p \wedge q \Leftrightarrow p \vee \sim q$ must be T.

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$\sim p \wedge q$	$\sim p \wedge q \Leftrightarrow p \vee \sim q$
F	F	T	T	F	F	T

The answer is T.

5) Answer: C

Explanation: Note that if two statements are logically equivalent, then the biconditional of these statements is a tautology. Hence, option C is the only right answer, and all options have a false statement.