In the previous chapter, we've refreshed our minds with logical propositions including their conjunction, disjunction, negation, implication, and biconditional. You have also learned the converse, inverse, and contrapositive of conditional statements. Our discussion focused on these concepts, especially on their truth values (i.e. whether they are true or false).

We identified the truth values of those statements just by analyzing them. However, it becomes tedious to identify the truth values of statements especially if they are too long or complex. For this reason, we must have a reliable and convenient method to identify truth values.

In this reviewer, we introduce to you how to use truth tables - a powerful tool in Propositional logic. Also, you will learn what are logical equivalences and how truth tables can be used to describe them.

Truth Value of a Proposition: A review

We have discussed with the previous reviewer that a proposition can only have one truth value. That is, a proposition can either be true (T) or false (F). The **Law of Excluded Middle** tells us that it is impossible for a proposition to be both true and false at the same time.

A proposition is true if it states a piece of true or correct information and false otherwise. Meanwhile, if we are dealing with complex propositions formed using connectives such as conjunctions ($\wedge$), disjunctions ($\vee$), negation ($\Rightarrow$), conditional statement (or implication) ($\Rightarrow$), and biconditional statement ($\Leftrightarrow$), the truth value of the statement depends on the truth values of the respective propositions that composed it. For instance, the truth value of p ⇒ q depends on the respective truth values of p and q.

To refresh our minds about how to identify the truth value of conjunctions, disjunctions, negations, conditional statements, and biconditionals, we summarize what we have learned again the previous reviewer in the table below:

Connective	**Truth Value**
Conjunction (p ∧ q)	True if both p and q are true
Disjunction (p ∨ q)	True if at least one of p and q are true
Negation (~p)	False if p is true True if p is false
Conditional Statement (p ⇒ q)	False if p is true and q is false. Otherwise, true
Biconditional Statement (p ⇔ q)	True if p and q have the same truth values

Now that we have warm-up our minds with what we have learned from the last reviewer, let us proceed with the actual topic we have for this chapter.

Truth Tables

We have stated earlier that the truth value of complex propositions formed by connectives depends on the individual truth values of its components. Actually, we can easily show how the truthfulness or falsity of a statement depends on its components using **truth tables**.

Let us start with the simplest case: the negation of a statement. Shown below is an example of a truth value for p and its negation ~p.

p	~p
T	F
F	T

What's good with the truth table above is that it is very intuitive. Clearly, the table states that if p is a true proposition, then ~**p** is false and vice versa.

Now, let us create a truth table for p ∧ q:

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

Let us analyze the table above:

The first row of the table shows the propositions p, q, and the conjunction p ∧ q. We have already learned that a conjunction p ∧ q is true if both p and q are true, then as seen in the table, only row 2, where both p and q are true, shows a "true" truth value for the conjunction. The remaining rows provide a "false" truth value since in those rows (rows 3 to 5), p and q are not both true.

Now that you have an understanding of truth tables and what they present. The next question we have to answer is: *How to construct a truth table?*
We will try to answer this question in the next section

Constructing truth tables

To construct a truth table for logical connectives (conjunction, disjunction, negation, etc.), we can follow the steps indicated below:

1) **Determine how many propositions are involved in the statement**
   Let *n* be that number of propositions in the statement.
2) **Construct a table with 2^n rows to put the possible truth values of the propositions**
   Of course, you also have to include one additional row on top of the 2^n rows to put the names of the propositions.
3) **Assign all possibilities of truth values of each proposition.**
   We will show later in our example how to perform this.
4) **Use your knowledge about the truth values of connectives to identify the truth value for the column dedicated to the connective**

Let us use this step to recreate the truth table for the conjunction of P and Q which we have presented in the previous section.

Example: Create a truth table for $P \wedge Q$

1) **Determine how many propositions are involved in the statement**
   We have 2 propositions, P and Q. Thus, we have n = 2.
2) **Construct a table with 2^n rows to put the possible truth values of the propositions**
   Since we have n = 2, then we have to prepare a table with $2^n = 2^{(2)} = 2 \times 2 = 4$ rows. We also need to include one additional row on top of these 4 rows so that we can put the proposition labels. Hence, we have a table with 5 rows.

P	Q	P ∧ Q

Now, we provided 3 columns for the table since we have to put proposition P, proposition Q, and the conjunction of interest P ∧ Q.

3) Assign all possibilities of truth values of each proposition.

P	Q	P ∧ Q

We have to put truth values to the columns dedicated for the propositions (P and Q). Actually, it's totally up to you how to assign truth values for them. However, to make it easier and more organized, start with the first column pertaining to P. An easy way to fill this column is by putting two "true" (T) and two "false" (F) (since there are 4 columns, we just divide it by 2 so that the T's and F's have equal numbers).

P	Q	P ∧ Q
T		

T		
F		
F		

For the second column, we will assign truth values in a manner where no combination will be repeated. In our case above, it is best to assign T's and F's for proposition Q in alternating manner.

P	**Q**	**P ∧ Q**
T	T	
T	F	
F	T	
F	F	

4) **Use your knowledge about the truth values of connectives to identify the truth value for the column dedicated to the connective.**

P	**Q**	**P ∧ Q**
T	T	
T	F	
F	T	

F	F	

Since we know that a conjunction will be true only if both propositions are true, then we only put T in the second row because P and Q are both true in this row. For the other rows, we just put F since not both P and Q are true on those rows.

P	**Q**	**P ∧ Q**
T	T	T
T	F	F
F	T	F
F	F	F

That's it, we have made a truth table for conjunction.

You can try using the steps we have discussed in this section to create the respective truth tables for disjunction, conditional statements, and biconditional statements. We will just present their respective truth tables below since it will be too long if we will discuss them one by one.

Truth table for Disjunction

P	**Q**	**P ∨ Q**
T	T	T
T	F	T
F	T	T

F	F	F

Truth table for Conditional Statement (or logical implication / “If and then” statement)

P	Q	P ⇒ Q
T	T	T
T	F	F
F	T	T
F	F	T

Truth table for Biconditional Statement

P	Q	P ⇔ Q
T	T	T
T	F	F
F	T	F
F	F	T

We will use our knowledge about truth tables to identify the truth values of various complex statements.

Applying the truth tables

Let us try to create a truth table for $P \lor Q \Rightarrow R$ using the steps we have discussed in the previous section.

1) **Determine how many propositions are involved in the statement**
In the given statement $P \lor Q \Rightarrow R$. There are three propositions involved: P, Q, and R. So, we have n = 3.

2) **Construct a table with 2^n rows to put the possible truth values of the propositions**
Since we have n = 3, then we need to prepare a table with $2^3 = 8$ rows plus one additional row where we can put the names of the propositions involved.

P	**Q**	**R**	$P \lor Q$	$P \lor Q \Rightarrow R$

You might be wondering why I have made a table with 5 columns. There are 5 columns since there are three individual propositions (P, Q, and R) and there are two connectives involved (disjunction and implication).

3) Assign all possibilities of truth values of each proposition.
We will start by filling the first column that pertains to proposition P. Since we have 8 rows below column P, we need to put four T's and four F's in this column.

P	Q	R	$P \vee Q$	$P \vee Q \Rightarrow R$
T				
T				
T				
T				
F				
F				
F				
F				

For the second column, we fill it by putting alternating T and F.

P	Q	R	$P \vee Q$	$P \vee Q \Rightarrow R$
T	T			
T	F			
T	T			
T	F			
F	T			

F	F			
F	T			
F	F			

You might notice that there are repeated combinations (row 1 and row 3), don't sweat it because we have not yet assigned truth values for R.

It is important how you will fill the third column since you must avoid repeating any combination. It's totally up to you how to fill this column as long as no combination is repeated.

Here's a suggestion, you can fill the third column by alternating two T's and two F's. Doing this will fill column "R" without repeating any combination.

P	**Q**	**R**	$P \vee Q$	$P \vee Q \Rightarrow R$
T	T	T		
T	F	T		
T	T	F		
T	F	F		
F	T	T		
F	F	T		
F	T	F		
F	F	F		

Are there any repeated combinations? None! We're good to go to the next section.

4) Use your knowledge about the truth values of connectives to identify the truth value for the column dedicated to the connectives

P	**Q**	**R**	$P \vee Q$	$P \vee Q \Rightarrow R$
T	T	T		
T	F	T		
T	T	F		
T	F	F		
F	T	T		
F	F	T		
F	T	F		
F	F	F		

Now, we're on the fourth and fifth column which involve connectives.

Let us start with the fourth column which is the disjunction of P and Q. Note that a disjunction will be true if at least one of the propositions is true. Hence, $P \vee Q$ is true if at least one of P and Q is true. Rows 2, 4, 5, 6, and 8 show at least one true proposition between P and Q. The remaining rows show neither true proposition so we put F to them.

P	Q	R	$P \lor Q$	$P \lor Q \Rightarrow R$
T	T	T	T	
T	F	T	F	
T	T	F	T	
T	F	F	T	
F	T	T	T	
F	F	T	F	
F	T	F	T	
F	F	F	F	

We're now on the last column that pertains to $P \lor Q \Rightarrow R$. To identify the truth values for each row under this column, we have to refer to the 4th column as the hypothesis and the 3rd column as the conclusion.

Note that we have to put F on rows 4, 5, and 8 since they show a true hypothesis and a false conclusion. Meanwhile, the remaining rows must be true.

P	Q	R	$P \lor Q$	$P \lor Q \Rightarrow R$
T	T	T	T	T
T	F	T	F	T
T	T	F	T	F
T	F	F	T	F
F	T	T	T	T

F	F	T	F	T
F	T	F	T	F
F	F	F	F	T

That's it, we have completed a truth table for $P \vee Q \Rightarrow R$.

Using the truth table above, can you identify the truth value of $P \vee Q \Rightarrow R$ if P is true, Q is false and R is false?

P is true, Q is false, and R is false on row 5. In row 5, we can see that the truth value of $P \vee Q \Rightarrow R$ is F. Hence, the answer to this question is F.

Example: Identify the truth value of $P \Rightarrow Q \wedge \sim R$ **if P is true, Q is false, and R is false.**

We can use a truth table to answer this. However, we don't have to create an entire truth table just to answer this since the problem already specified the truth values of each proposition.

We have three propositions (P, Q, and R) and three connectives involved (implication, conjunction, and negation). So, we prepare a table with 6 columns and 2 rows (1 for labels and 1 for truth values.

P	**Q**	**R**	$\sim R$	$Q \wedge \sim R$	$P \Rightarrow Q \wedge \sim R$

It was stated in the problem that P is true, Q is false, and R is false. We put these truth values in the second row:

P	**Q**	**R**	$\sim R$	$Q \wedge \sim R$	$P \Rightarrow Q \wedge \sim R$

T	F	F			

We use our knowledge about connectives to fill the remaining columns. For the fourth column, we only need to identify the truth value of the negation of R. Since R is F, then ~R must be T.

P	**Q**	**R**	$\sim R$	$Q \wedge \sim R$	$P \Rightarrow Q \wedge \sim R$
T	F	F	T		

Now, $Q \wedge \sim R$ must be false since not both Q and $\sim R$ are true (Q is false). Hence, we put F for the fifth column.

P	**Q**	**R**	$\sim R$	$Q \wedge \sim R$	$P \Rightarrow Q \wedge \sim R$
T	F	F	T	F	

Lastly, we have to fill the rightmost column. $P \Rightarrow Q \wedge \sim R$ means that we have P as the hypothesis and $Q \wedge \sim R$ as the conclusion. Based on the table above, P is true while $Q \wedge \sim R$ is false. This means that we have a true hypothesis and a false conclusion, implying that the conditional statement must be false. Hence, we put F in the last column.

P	**Q**	**R**	$\sim R$	$Q \wedge \sim R$	$P \Rightarrow Q \wedge \sim R$
T	F	F	T	F	F

Therefore, the truth value of $P \Rightarrow Q \wedge \sim R$ if P is true, Q is false, and R is false, is false.

Example: Is Q ∨ ~P ⇔ ~R ∧ Q true if P, Q, and R are all true?

The statement $Q \vee \sim P \Leftrightarrow \sim R \wedge Q$ is extremely complicated to look at. But, with the help of truth tables, we can easily identify its truth value given that P, Q, and R are all true. Again, we don't have to create an entire truth table in this case since we are already interested in the truth value of the given statement if all P, Q, and R are true.

Let's start by constructing a table. We have 3 propositions involved (P, Q, and R) and five connectives involved (disjunction, biconditional, negation, and conjunction). Hence, we will have an 8-column table. We put T for P, Q, and R since this fact is given in the problem.

P	Q	R	~P	$Q \vee \sim P$	~ R	~R ∧ Q	$Q \vee \sim P \Leftrightarrow \sim R \wedge Q$
T	T	T					

Since P is T, then ~P is F. Also, since R is T, we can also conclude that ~ R is F.

P	Q	R	~P	$Q \vee \sim P$	~ R	~R ∧ Q	$Q \vee \sim P \Leftrightarrow \sim R \wedge Q$
T	T	T	F		F		

Since Q is T and ~P is F. Then, $Q \vee \sim P$ must be T since at least one of the propositions is true.

P	Q	R	~P	$Q \vee \sim P$	~ R	~R ∧ Q	$Q \vee \sim P \Leftrightarrow \sim R \wedge Q$
T	T	T	F	T	F		

Since ~ R is F and Q is T, then ~R ∧ Q is F since not both of the propositions are true.

P	Q	R	~P	$Q \lor \sim P$	~ R	~R ∧ Q	$Q \lor \sim P \Leftrightarrow \sim R \land Q$
T	T	T	F	T	F	F	

Lastly, to identify whether $Q \lor \sim P \Leftrightarrow \sim R \land Q$ is true or not. We look at the truth values of $Q \lor \sim P$ and $\sim R \land Q$. We can see from the table that $Q \lor \sim P$ is T while $\sim R \land Q$ is F. Recall that in a biconditional statement, if the statements on the left and on the right of the arrow are not of the same truth value, then the biconditional is F. Hence, $Q \lor \sim P \Leftrightarrow \sim R \land Q$ must be false.

P	Q	R	~P	$Q \lor \sim P$	~ R	~R ∧ Q	$Q \lor \sim P \Leftrightarrow \sim R \land Q$
T	T	T	F	T	F	F	F

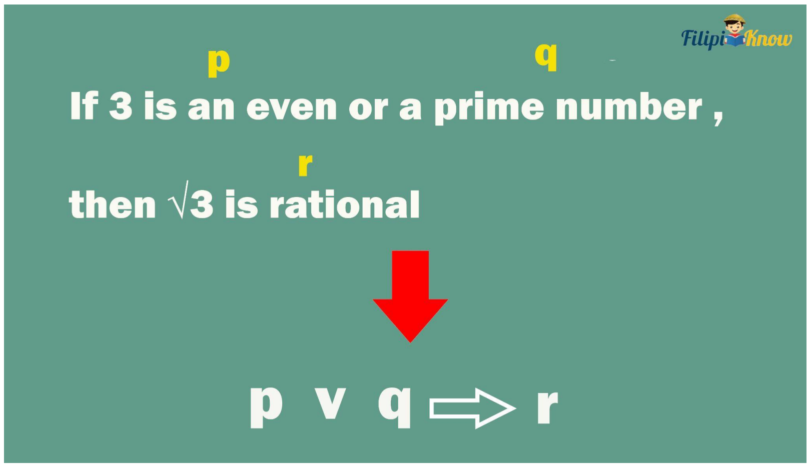
Therefore, if P, Q, R are all true, then $Q \lor \sim P \Leftrightarrow \sim R \land Q$ is false.

Example: Identify the truth value of "If 3 is an even or a prime number, then $\sqrt{3}$ is rational"

Let us start by assigning propositional variables for each proposition in the given statement:

Let *p* be the proposition "3 is an even number". This is a false proposition since 3 is an odd number.
Let *q* be the proposition "3 is a prime number". This is a true proposition. The only factors of 3 are 3 and itself.
Let *r* be the proposition "$\sqrt{3}$ is rational". This is false. There's no whole number that when multiplied by itself will result in 3.

Let us translate the statement *"If 3 is an even or a prime number then $\sqrt{3}$ is rational"* into a statement with propositional variables *p, q,* and *r*.



By translating, we will obtain $p \vee q \Rightarrow r$.

Let us create a truth table to identify whether $p \vee q \Rightarrow r$ is true or false. Recall that we have stated earlier that p is false, q is true, and r is false.

p	q	r	$p \vee q$	$p \vee q \Rightarrow r$
F	T	F		

Since p is F and q is T, then the conjunction $p \vee q$ must be T since one of p and q is true.

p	q	r	$p \vee q$	$p \vee q \Rightarrow r$
F	T	F	T	

In $p \vee q \Rightarrow r$, the hypothesis is $p \vee q$, which is T based on the truth table above. Meanwhile, the conclusion, which is *r*, is false. So, we have a true hypothesis and a false conclusion in *p v q ⇒ r*. Hence, *p v q ⇒ r* is false.

p	*q*	*r*	*p v q*	*p v q ⇒ r*
F	T	F	T	

The answer is false.

Tautology, Contradiction, and Contingency

A **tautology** is a statement that is always true regardless of the truth value of the propositions that composed it. In other words, a tautology is always whatever truth value you will assign to the propositions.

For instance, the statement p ∨ ~p is a tautology since it is always true regardless of the truth value assigned to p and ~p.

` Shown below is the truth value of p ∨ ~p:

p	~p	p ∨ ~p
T	T	T
F	T	T

Notice that whatever the truth values of p and ~p, p ∨ ~p is a true statement. Hence, p ∨ ~p is an example of a tautology.

What is a real example of p ∨ ~p which is a tautology? Consider p as the proposition “3 is an odd number”. Then, ~p is “3 is an even number”. Hence p ∨ ~p is “3 is an odd or even number". The statement "3 is an odd or even number" is a tautology since this statement is always true.

The opposite of tautology is a **contradiction.** A contradiction is a statement that is always false regardless of the truth value of the propositions that composed it.

For instance, the statement $p \wedge \sim p$ is an example of contradiction. This means that whatever the truth value of p and $\sim p$, the truth value of $p \wedge \sim p$ is always false.

Here's the truth table of $p \wedge \sim p$. See how $p \wedge \sim p$ is always false.

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

If a proposition is neither a tautology nor a contradiction, the proposition is called a **contingency**. This means that its truth value varies depending on the truth values of the propositions that composed it.

For example, $p \Rightarrow \sim p$ is a contingency since its truth value varies. Refer to its truth table below and observe its truth values.

p	$\sim p$	$p \Rightarrow \sim p$
T	F	F
F	T	T

Note that the truth value of P ⇒ ~P is not always true or not always false. Hence, P ⇒ ~P is a contingency.

Example: Determine if the following is a tautology, contradiction, or a contingency.

(a) (P ∧ Q) ⇒ P
(b) P ∨ ~Q ⇔ Q

We can answer this problem by creating truth tables for each statement:

(a) For *(P ∧ Q) ⇒ P*:

P	Q	*P ∧ Q*	*(P ∧ Q) ⇒ P*
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Based on the truth table above, the truth values of *(p ∧ q) ⇒ p* is always true regardless of the truth values of *p* and *q*. Hence, *(p ∧ q) ⇒ p* is a tautology.

(b) For P ∨ ~Q ⇔ Q

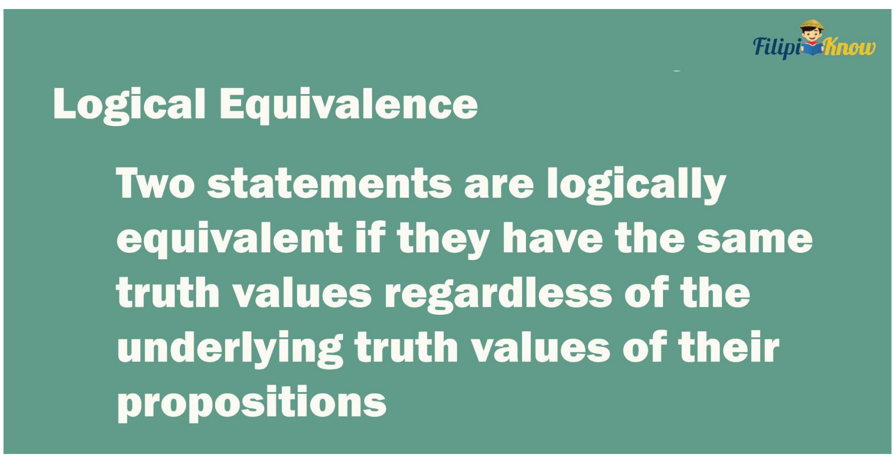
P	Q	~Q	P ∨ ~Q	P ∨ ~Q ⇔ Q
T	T	F	T	T
T	F	T	T	F

F	T	F	F	F
F	F	T	T	F

Based on our truth table above, the truth values of $p \vee \sim q \Leftrightarrow q$ varies (it is not always true or not always false). Hence, P ∨ ~Q ⇔ Q is considered as a contingency.

Logical Equivalence

Two statements are **logically equivalent** if they always have the same truth value. In other words, logical equivalent statements will produce similar truth values regardless of the truth values of the propositions that composed these statements.



Formally: *two statements m and n are logically equivalent (In symbols, m ≡ n) if m ⇔ n is a tautology.*

To make the concept of logical equivalence much clearer to you, consider these statements:

Statement 1 : $P \Rightarrow Q$
Statement 2: $\sim Q \Rightarrow \sim P$

Let us create the respective truth tables of each statement:

Statement 1 truth table:

P	**Q**	**$P \Rightarrow Q$**
T	T	T
T	F	F
F	T	T
F	F	T

Take note of the truth values of $P \Rightarrow Q$.

Statement 2 truth table

P	**Q**	**$\sim P$**	**$\sim Q$**	**$\sim Q \Rightarrow \sim P$**
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Now, compare the truth values of $P \Rightarrow Q$ and $\sim Q \Rightarrow \sim P$.

Truth values of $P \Rightarrow Q$	***Truth values of*** $\sim Q \Rightarrow \sim P$
T	T
F	F
T	T
T	T

Note how the truth values of the statements are the same for every assigned combination of truth values of P and Q .

This means that $P \Rightarrow Q$ and $\sim Q \Rightarrow \sim P$ are logically equivalent.

What have you noticed about $P \Rightarrow Q$ *and* $\sim Q \Rightarrow \sim P$*?* Yes, they are contrapositives! Here's an interesting fact: The contrapositive of a conditional statement and the conditional statement itself are logically equivalent. This also means that a conditional statement is true if its contrapositive is true. This property is known as the **Law of Contraposition.**

Example: Alice said that "If I live in Manila, then I live in the Philippines". Celine heard this and tried to restate it as "If Alice doesn't live in the Philippines, then she does not live in Manila". **Are Alice and Celine's statements logically equivalent?**

Yes, since Celine's statement is just the contrapositive of Alice's statement. Note that a conditional statement and its contrapositive are logically equivalent.

Example: Are $P \Rightarrow Q$ ***and*** $\sim P \vee Q$ ***logically equivalent?***

We use truth tables to show whether $P \Rightarrow Q$ and $\sim P \vee Q$ are logically equivalent:

Truth table for $P \Rightarrow Q$:

P	**Q**	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth table for $\sim P \vee Q$:

P	**Q**	$\sim P$	$\sim P \vee Q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Comparing the truth tables above, we can see that $P \Rightarrow Q$ and $\sim P \vee Q$ have the same truth values for every assigned combination of truth values of *P* and *Q* . For this reason, we conclude that $P \Rightarrow Q$ and $\sim P \vee Q$ are logically equivalent. In symbols: $P \Rightarrow Q \equiv \sim P \vee Q$

We have already shown that $P \Rightarrow Q$ and $\sim P \vee Q$ are logically equivalent. Let us provide a concrete example of a statement that represents them.

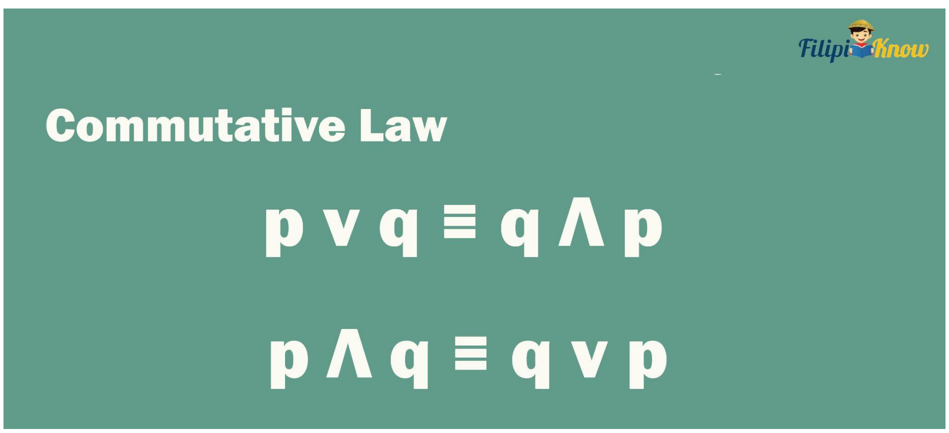
Let P be the proposition "Fred knows algebra" and Q be the proposition 'Fred is a high school student". Then, $P \Rightarrow Q$ is translated as "If Fred knows algebra, then he is a high school student" and $\sim P \vee Q$ is translated as "Fred does not know algebra or he is a high school student".

Since, we have shown earlier that $P \Rightarrow Q \equiv {\sim}P \vee Q$. Then, the statements: “If Fred knows algebra, then he is a high school student” and “Fred does not know algebra or he is a high school student” are logically equivalent. This means that if the statement “If Fred knows algebra, then he is a high school student” is true then the statement “Fred does not know algebra or he is a high school student” is also true.

Properties of Logical Equivalence

In this section, we will discuss the important properties of logical equivalence that can help us to identify whether two given statements are logically equivalent or not without the use of truth tables.

Commutative Law



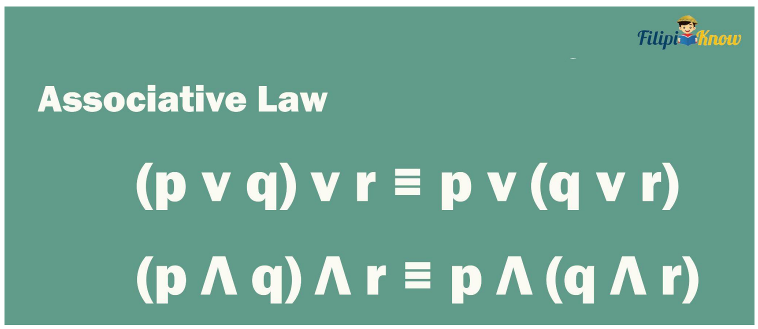
The commutative law is pretty easy to understand. It also states that the order of propositions in a conjunction and a disjunction is not important.

For instance, the statement "Jessie is a French and a college student" is logically equivalent with the statement "Jessie is a college student and a French".

This means that if the statement "Jessie is a French and a college student" is true, then the

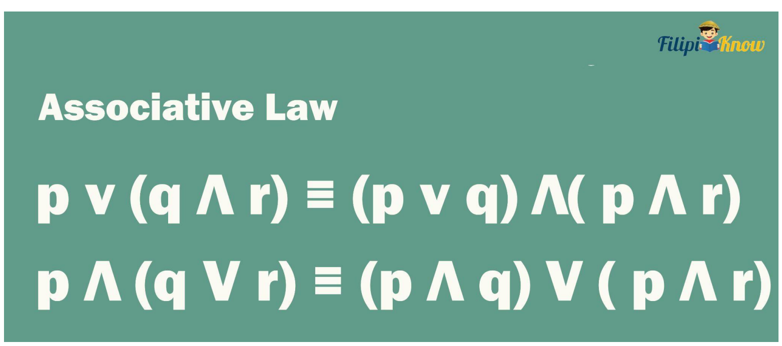
the statement "Jessie is a college student and a French " will also be true. Similarly, if "Jessie is a French and a college student" is a false statement, then the statement "Jessie is a college student and a French" is also false.

Associative Law



Basically, this property tells us that in conjunction and disjunction, the groupings of the propositions do not matter and will always yield the same truth values. This means that (p v q) v r will have identical truth values with p v (q v r) (You can try proving this using truth tables).

Distributive Law

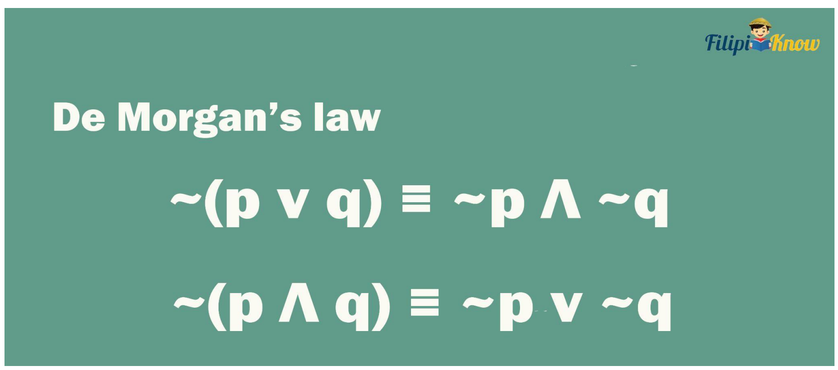


The distributive law of logical equivalence allows us to distribute an outer connective to an inner connective.

For instance, in p v (q v r), we distribute “p v” to (q v r) to obtain (p v q) ∧ (p v r). Note that in between (p v q) and (q v r), we have to put the opposite of “v” (disjunction symbol) which is the “∧” (conjunction symbol).

Similarly, we can distribute “p ∧” to (q ∧ r) to obtain (p ∧ q) v (p ∧ r). We have to put the opposite of “∧” (conjunction symbol), which is “v” (disjunction symbol) in between the group.

De Morgan’s law



This law states that the negation of the disjunction of two statements is logically equivalent to the conjunction of the negation of each statement. Moreover, it also states that the negation of the conjunction of two statements is logically equivalent to the disjunction of the negation of each statement.

For instance, consider the statement “The building will be demolished or sold”. According to De Morgan’s law, the negation of this statement is “The building will not be demolished and will not be sold”.

Another example: suppose the statement “*x* is a whole number and an integer”. According to De Morgan’s law, the negation of this statement is “*x* is not a whole number nor an integer”.

Example: Use De Morgan's law to construct the opposite or negation of the statement "It is prohibited to bring pets or sharp objects in the hotel".

Solution: *"It is allowed to bring pets and sharp objects in the hotel"*

Equivalence of an implication and a contrapositive law

Equivalence of an implication and a contrapositive

$$p \Rightarrow q \equiv \sim q \Rightarrow \sim p$$

Also known as the law of contraposition, this rule states that given a conditional statement or a logical implication, its contrapositive is logically equivalent to itself.

For instance, the statement "If *p* is an integer, then it is a rational number" is logically equivalent to its contrapositive "If *p* is not a rational number, then *p* is an integer". This implies that if the first statement is true, then the second statement (the contrapositive) is also true (same case if false)

Negation of an Implication

Negation of an implication

$$\sim(p \Rightarrow q) \equiv p \land \sim q$$

According to this law, the negation of a logical implication (or a conditional statement) is logically equivalent to the conjunction of its hypothesis and the negation of its conclusion.

For instance, suppose we want to negate the conditional statement *"If I write a poem, then I'm inspired".* We can form it by simply constructing the conjunction of the hypothesis "I write a poem" and the negation of the conclusion "I'm not inspired". Hence, the negation of the given conditional statement is "*I write a poem but I'm not inspired".*

Idempotent laws

Idempotent laws

$$p \vee p \equiv p$$

$$p \wedge p \equiv p$$

This law states that the conjunction of a statement to itself is logically equivalent to the statement itself. Similarly, the disjunction of a statement to itself is logically equivalent to the statement itself.

Implication to disjunction

Implication to disjunction

$$p \Rightarrow q \equiv {\sim}p \wedge q$$

The last logical equivalence rule that we're going to discuss is the property of a logical implication or a conditional statement to be written into a logically equivalent disjunction.

This law states that a conditional statement or logical implication is logically equivalent to the disjunction of negation of the hypothesis and the conclusion.

For instance, consider the statement "If you obtained a 95% in the exam, you will be the class' valedictorian". We can rewrite this conditional statement into an equivalent disjunction of the negation of the hypothesis "you obtain a 95% in the exam" and the conclusion "you will be the class valedictorian". Hence, the given conditional statement can be rewritten as a disjunction this way: "You will not obtain a 95% in the exam or you will be the class valedictorian".

You might be wondering how this disjunction "You will not obtain a 95% in the exam or you will be the class valedictorian" becomes logically equivalent to "If you obtained a 95% in the exam, you will be the class' valedictorian" Well, analyze the disjunction carefully. It states that either you will not obtain 95% on an exam or you will be a class valedictorian. This implies that if it didn't happen that you did not obtain a 95% in the exam, then it means that you get a 95% in the exam and you are the valedictorian. This is exactly what the given conditional statement states that you will be a class valedictorian if you obtain a 95% in the exam.

We summarize the logical equivalence rules we have discussed in the table below:

Logical Equivalence laws	
Commutative law	$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$
Associative law	$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive law	$p \vee (q \vee r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (q \wedge r)$
De Morgan's law	$\sim(p \vee q) \equiv \sim p \wedge \sim q$ $\sim(p \wedge q) \equiv \sim p \vee \sim q$
Equivalence of an implication and a contrapositive	$p \Rightarrow q \equiv \sim q \Rightarrow \sim p$
Negation of an Implication	$\sim(p \Rightarrow q) \equiv p \wedge \sim q$

Idempotent laws	p ∨ p ≡ p p ∧ p ≡ p
Implication to disjunction law	p ⇒ q ≡ ~p ∧ q