PMA Mathematics Reviewer Answer Key

Set 3: Geometry and Trigonometry

1) Answer: D
Explanation: The measure of an interior angle of a regular polygon is given by the formula m = $\frac{180(n-2)}{n}$ where *n* is the number of sides of the regular polygon.

A regular hexagon has 6 congruent sides. Hence, to determine the measure of an interior angle of a regular hexagon, we substitute *n = 6* to the formula above.

m = $\frac{180(n-2)}{n}$

m = $\frac{180(6-2)}{6}$

m = $\frac{180(4)}{6} = 720/6 = 120$

Therefore, the measure of an interior angle of a regular hexagon is 120 degrees.

2) Answer: B
Explanation: The perimeter of a triangle is given by the formula P = a + b + c where a, b, and c are the measurements of its sides. Since we have an isosceles triangle in the problem, the two sides should be congruent. Hence, we can assume that both a and b are equal to 25.5 cm. Meanwhile, we assume *c* as the measurement of the side that is not congruent to the two (or the remaining side).

The perimeter of the isosceles triangle is 70. Thus, we have this equation:

P = a + b + c	
70 = 25.5 + 25.5 + c	*Substitution*
70 = 51 + c	
-51 + 70 = c	*Transposition method*
19 = c	
c = 19	*Symmetry property of equality*

Thus, the remaining side of the isosceles triangle measures 19 cm.

3) Answer: C
Explanation: In a right triangle, the sum of the squares of the measurements of its legs is equal to the square of the measurement of its longest side (or hypotenuse). This is known as the Pythagorean theorem.

In symbols, $c^2 = a^2 + b^2$, where *c* is the longest side and *a* and *b* are the legs.

According to the given problem, the measurements of the legs are 6 meters and 9 meters. Hence, we have a = 6 and b = 9. Let's now solve for c:

$c^2 = a^2 + b^2$
$c^2 = (6)^2 + (9)^2$ *Substitution*
$c^2 = 36 + 81$
$c^2 = 117$
$\sqrt{c^2} = \sqrt{117}$ *Take the square root of both sides*
$c = \sqrt{117}$

Thus, the measurement of the longest side is $\sqrt{117}$ meters.

4) Answer: B
Explanation: Let *w* be the width of the rectangle. Since the length is 18 units longer than the width, then we can express the measurement of the length as *w + 18*.

The perimeter of the rectangle is 104 units. If the formula for the perimeter of a rectangle is P = 2l + 2w, then we have

$P = 2l + 2w$
$104 = 2(w + 18) + 2w$ *Since the perimeter is 104 units*

Let us now solve for *w* or width and then identify the measurement of the length to compute for the area of the rectangle.

104 = 2(w + 18) + 2w	
104 = 2w + 36 + 2w	*Distributive property*
104 = 4w + 36	*Combining like terms*
-36 + 104 = 4w	*Transposition method*
68= 4w	
4w = 68	*Symmetry property of equality*
4w/4 = 68/4	
w = 17	

From our calculation above, the measurement of the width is 17 meters. Since the length is 18 units longer than the width, then the length is 17 + 18 = 35 meters long.

Let us now compute the area by multiplying the measurements of the length and width of the rectangle.

A = length x width
A = 18 x 35
A = 630

Thus, the area of the rectangle is 630 square units.

5) Answer: D
Solution: Let *x* be the measurement of the smaller angle. Since the larger angle is 40 degrees larger than the smaller, then we can express the measurement of the larger angle as *x + 40*.

If these angles are supplementary, it means that the sum of the measurements of these angles is 180 degrees. Hence, we have this equation:

x + (x + 40) = 180	
2x + 40 = 180	*Combining like terms*

2x = -40 + 180
2x = 140 *Transposition method*
2x/2 = 140/2
x = 70

Since *x* is the measurement of the smaller angle, then the smaller angle measures 70 degrees.

6) Answer: A
Explanation: The formula for the area of an equilateral triangle is given by this formula:

$A = \frac{s^2\sqrt{3}}{4}$, where *s* is the measurement of a side of the equilateral triangle

By substituting s = 6 to the formula above:

$$A = \frac{s^2\sqrt{3}}{4}$$

$$A = \frac{(6)^2\sqrt{3}}{4}$$

$$A = \frac{36\sqrt{3}}{4} = \frac{36}{4}\sqrt{3} = 9\sqrt{3}$$

Therefore, the area of the equilateral triangle is $9\sqrt{3}$ cm^2

7) Answer: C
Explanation: The value of the sine function of the acute angle 60 degrees is equivalent to $\sqrt{3}$ / 2. We can derive this by knowing that in 30-60-90 right triangle, the opposite side of the 60-degree angle is $\sqrt{3}$ units long and the hypotenuse is 2 units long. Since, the sine function is defined as opposite side / hypotenuse, then sin 60 = $\sqrt{3}$ / 2. Hence, the value of *x* in sin x = $\frac{\sqrt{3}}{2}$ should be 60 degrees.

8) Answer: B

Explanation: To convert $\frac{3\pi}{2}$ radians to degrees, we have to multiply it by $\frac{180}{\pi}$:

$$\frac{3\pi}{2} \times \frac{180}{\pi} = \frac{540\pi}{2\pi} = 270$$

From our calculation above, we have obtained that the equivalent degree measurement of $\frac{3\pi}{2}$ radians is 270 degrees.

9) Answer: C

Explanation: Among the given choices, only option C indicates a correct mathematical sentence. Based on the given figure, the opposite side of the acute angle θ is 7 while the adjacent side is 24. Tangent is defined as the ratio of the lengths of the opposite side to the adjacent side. Therefore, the value of tan θ is indeed equal to 7/24.

10) Answer: B

Explanation: Recall that the perimeter of a triangle is just the sum of the measures of its three sides. Thus, the perimeter of a right triangle is just the sum of the lengths of its legs and hypotenuse

In the given problem, the sin of the acute angle θ of the given right triangle has a value of 10/26. Recall that the definition of the sin θ is the ratio of the length of the opposite leg to the length of the hypotenuse or:

$$sin\,\theta = opposite\ side\ /\ hypotenuse$$

Since sin θ = 10/26, then the opposite side of the right triangle measures 10 units and its hypotenuse measures 26 units. Now that we're able to reveal the two sides of the right triangle (one leg and the hypotenuse), we need one more side so that we can calculate the perimeter of the right triangle. To find that remaining side, we need to find the value or the measurement of the adjacent side of the acute angle θ.

Recall that by the Pythagorean theorem:

$(\text{Opposite side})^2 + (\text{adjacent side})^2 = (\text{hypotenuse})^2$

We already have opposite side = 10 and hypotenuse = 26; inputting these values to the equation above: $(10)^2 + (\text{adjacent side})^2 = (26)^2$

For the sake of conciseness, let x be the length of the adjacent side:

$(10)^2 + (x)^2 = (26)^2$

Let us now do the math and compute for the value of x in the equation above:

$(10)^2 + (x)^2 = (26)^2$

$100 + x^2 = 676$

$x^2 = -100 + 676$ *Transposition method*

$x^2 = 576$

Take the square root of both sides of equation

$\sqrt{x^2} = \sqrt{576}$

$x = \pm 24$

We reject the negative value of x since x represents the length of the adjacent side. Hence, we will take x = 24 only

Thus, the length of the adjacent side is 24 units.

We now have three sides:
- Opposite side = 10 units
- Adjacent side = 24 units
- Hypotenuse = 26 units.

We add the measurements of these sides to obtain the perimeter of the right triangle: P = 10 + 24 + 26 = 60 cm.

Thus, the perimeter of the right triangle is 60 units