*Have you ever used a protractor?*

If we want to draw an angle, we need a protractor and use it as a guide. Through the protractor, we can provide a visual depiction of the concept of angles.

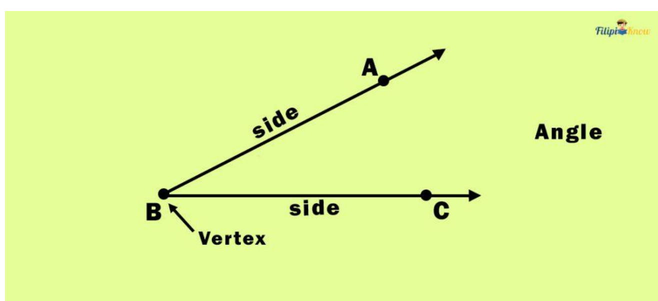
*But what exactly is an angle? Why do we have to draw them using protractors? What is their practical significance?*

Angles are of particular importance in the study of geometry. They possess powerful characteristics that enable us to understand geometric figures better.

In this module, you will learn everything you must remember about angles, including their definition, classification, and properties.

## What Is an Angle?

An angle consists of two rays that have the same endpoint. The endpoint where the rays intersect is called the **vertex.** Meanwhile, the rays are called the **sides**.

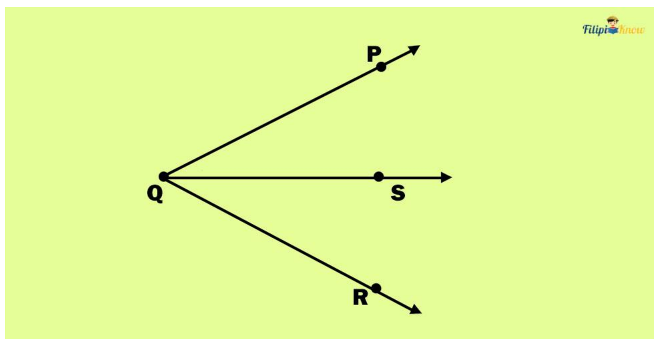



We use the points of the sides of the angles to name the angle. In the figure above, we call the angle ∠ABC where B is the angle's vertex (since it is the common endpoint of the rays). We can also name the angle ∠CBA.

Note that in naming an angle, we must put the letter representing the vertex in the middle.

Be careful when naming an angle. In the given figure above, we can call it ∠ABC or ∠CBA but not ∠BAC nor ∠CAB since the vertex (point B) must always be in the middle of the angle's name.

**Sample Problem 1:** Determine the angles you can see in the given figure below.




**Answer:** The angles are ∠PQR, ∠PQS, and ∠SQR.

**Sample Problem 2:** Using the same figure above, is SRQ an angle?

**Answer**: No, because there are no two rays with R as their common endpoint.

## Angle Measurement

Just like any geometric figure, we can also measure an angle.

To understand the "measurement of an angle", we need to learn the Protractor Postulate.

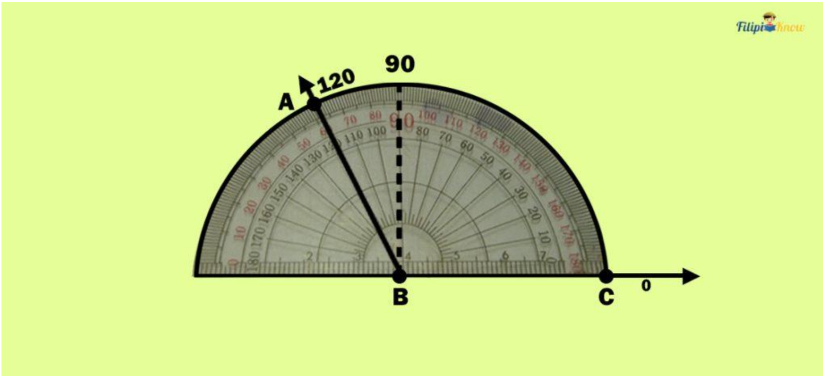
### Protractor Postulate

*"The measurement of angle refers to the measurement between two rays which can be designated as a real number from 0 to 180 degrees."*

The protractor postulate assumes that every angle can measure 0 to 180 degrees. Degrees (°) is the unit of measure we use for angles.

*How exactly do we measure an angle?*

We measure an angle using a protractor. Suppose the angle ∠ABC below that was placed with a protractor. We put the ABC vertex in the protractor's lower middle part. It is clearly seen that ray AB is pointed to 120° while point BC is pointed to 0°. The measurement of ∠ABC is the absolute value of the difference between the numbers that the rays are pointed to. Thus, the measure of ∠ABC is 120°



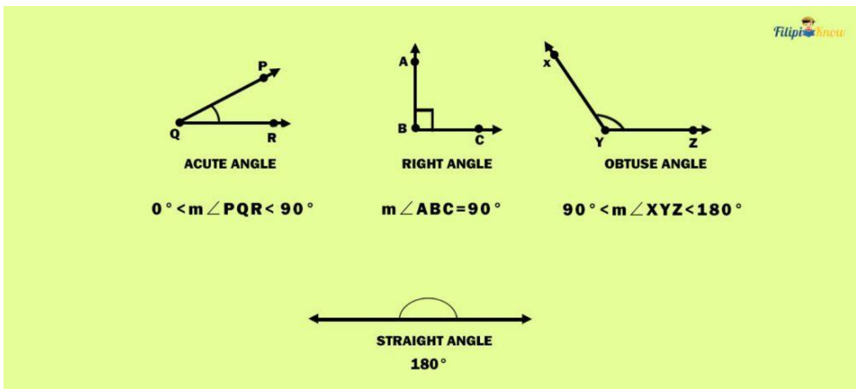

*We determine the degree measurement of an angle using a protractor*

It's nice to learn how to measure angles using protractors. However, since you are preparing for a college entrance exam (or civil service exam), it is most likely that the measurement of the angles is already given in the questions, so you don't have to use a protractor to determine the measure of an angle. We just have provided you with an idea of the measurement of the angle.

We use the symbol m∠ABC to refer to the measurement of angle ∠ABC. Hence, if ∠ABC measures 120°, then m∠ABC = 120°.

## Classification of Angles

We can classify angles according to their measurement. An angle can be an acute angle, a right angle, an obtuse angle, or a straight angle.




### 1. Acute Angle

An acute angle is an angle whose measure is between 0° and 90°.

For instance, if m∠PQR = 45°, then ∠PQR is an acute angle.

### 2. Right Angle

A right angle is an angle whose measure is exactly 90°. Notice how the right angle looks like the letter "L."

Take note of the word "exactly" in the definition of right angles. The term "exactly" implies that the measure of a right angle must be **strictly** 90 degrees. If the angle measure is 90.5°, we cannot consider it a right angle anymore.

### 3. Obtuse Angle

An obtuse angle is an angle whose measure is between 90° to 180°.

For instance, if m∠XYZ = 105°, then ∠XYZ is an obtuse angle.

### 4. Straight Angle

If an angle has a measurement of exactly 180°, then we call that angle a straight angle (which is a straight line also).

**Sample Problem 1:** Determine if the following angles are acute, right, obtuse, or straight.

1. m∠ABC = 125°
2. m∠COR = 90.5°
3. m∠RAW = 0.01°
4. m∠CDO = 179.12°

**Answer:**

1. Obtuse, since 125 is between 90 to 180
2. Obtuse, since 90.5 is between 90 to 180 (the angle is not a right angle)
3. Acute, since 0.01 is between 0 to 90
4. Obtuse, since 179.12 is between 90 to 180

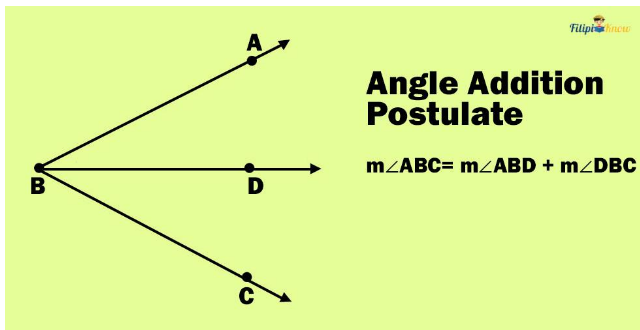
**Sample Problem 2:** What type of angle is formed by the hands of the clock when it's three o'clock?

**Answer:** It is obvious that the angle formed by the hands of the clock at three o'clock is a 90° angle since it resembles the letter "L." Thus, the angle formed is a right angle.

**Sample Problem 3:** What type of angle is formed by the hands of the clock when it's two o'clock?

**Answer:** In the previous example, we have concluded that when the hands of the clock are at three o'clock, then the hands of the clock form a right triangle. Since the angle formed when it's two o'clock is shorter than the angle formed when it's three o'clock, the angle must be an acute angle.

## Angle Addition Postulate

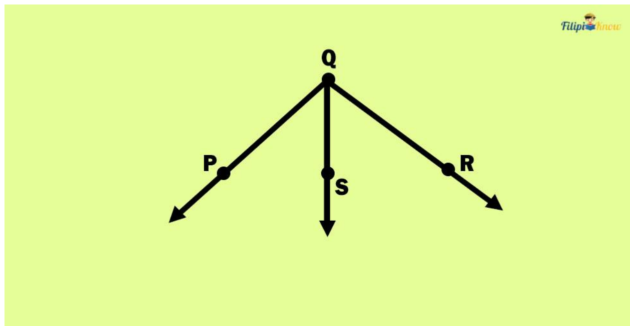



> *"If D is in the interior of ∠ABC, then the measure of ∠ABC is equal to the sum of the measures of ∠ABD and ∠DBC"*

In symbols, m∠*ABC* = *m*∠*ABD* + *m*∠*DBC*

The angle addition postulate is very intuitive and self-explanatory. The concept of the angle addition postulate is analogous to the concept of the segment addition postulate discussed in the previous reviewer. To find the measure of the entire angle ∠*ABC* in the figure above, we can just add the angles that composed it, which are ∠ABD and ∠*DBC*.

**Sample Problem 1:** If ∠PQR = 25, ∠PQS = 3x + 10, and ∠SQR = 2x, determine the value of x.




**Solution:**

The angle addition postulate tells us that the measure of the entire angle ∠*PQR* is equal to the sum of the measures of the angles that it contains (which are ∠PQS and ∠SQR).

Hence: m∠*PQR* = *m*∠*PQS* + *m*∠*SQR*

Using the values given in the problem:

m∠*PQR* = *m*∠*PQS* + *m*∠*SQR*

25 = (3x + 10) + 2x

We can now solve the value of *x* in the given equation above:

25 = (3x + 10) + 2x

25 = 5x + 10 *Combining like terms*

-10 + 25 = 5x *Transposition method*

15 = 5x

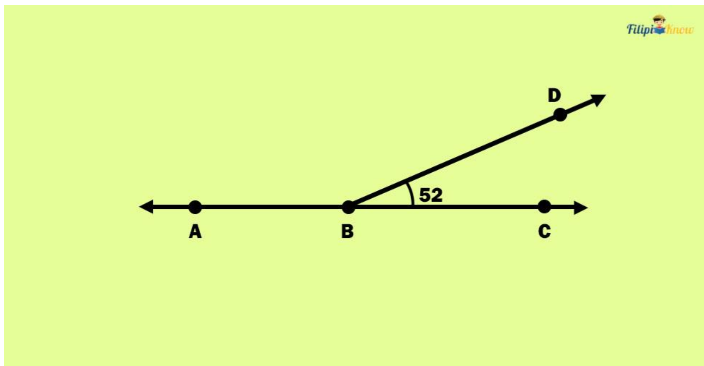
15/5 = 5x/5 *Dividing both sides by 5*

3 = x

x = 3 *Symmetric property of equality*

Thus, the value of *x* must be 3.

**Sample Problem 2:** Using the figure below, determine the measure of ∠ABD.




**Solution:**

The only given value is the measure of ∠*DBC* which is 52 degrees.

Notice that the angles ∠*ABD and* ∠*DBC* form a straight angle ∠*ABC.* We know that a straight angle has a measure of 180 degrees. Therefore, using the angle addition postulate:

m∠*ABC* = *m*∠*ABD* + *m*∠*DBC*

Since ∠*ABC* is a straight angle:

180 = *m*∠*ABD* + *m*∠*DBC*

Using the given value of *m*∠*DBC* which is 52:

180 = *m*∠*ABD* + *52*

Solving for the equation above:

-52 + 180 = *m∠ABD*

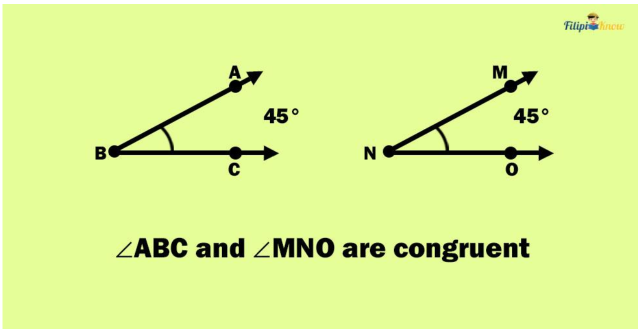
128 = *m∠ABD*

*m∠ABD* = 128

Thus, the measure of ∠*ABD* is 128°.

## Congruent Angles

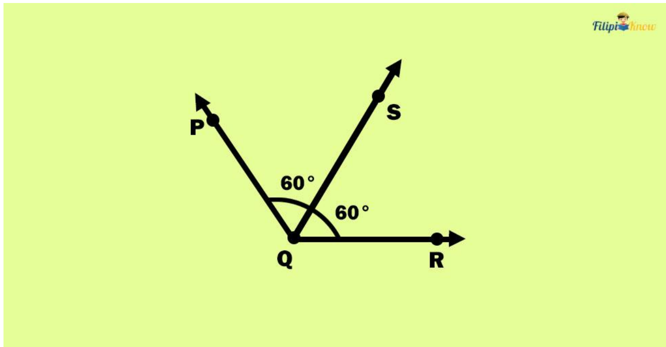
Angles are congruent if they have the same measurement. This means that congruent angles have the same form and size.




Do you still remember one of Euclid's postulates that states that all right angles are congruent?

It is pretty obvious why the postulate is true. We defined earlier that right angles have a degree measure that is **exactly** 90°. Thus, all right angles that you create will always have a degree measurement of 90°. Hence, it is logically sound to conclude that all right angles are congruent.

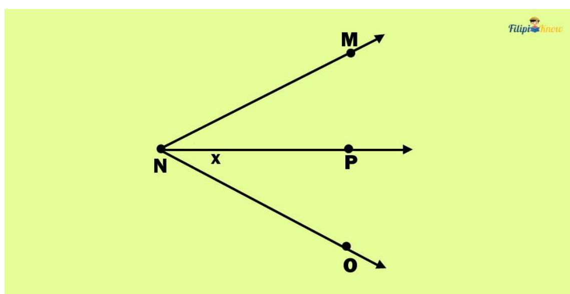
## Angle Bisector




In the figure above, ray QS divides ∠PQR into two congruent angles, namely ∠PQS and ∠SQR. Ray QS is an example of an **angle bisector**.

**An angle bisector is a ray that divides an angle into two congruent angles.**

**Sample Problem:** In the figure below, ∠MNO is 60°. If ray PN bisects angle ∠MNO, what is the measure of ∠ONP?




**Solution:**

*Method 1*

Since, ray PN bisects ∠MNO, then we can conclude that ∠*MNO* is divided into congruent angles, ∠MNP and ∠ONP. This implies that the measure of angles ∠MNP and ∠ONP are equal.

Therefore, to determine the measure of ∠ONP, we can just simply divide the measure of ∠MNO (which is 60 degrees) by 2 since the angles ∠*MNP* and ∠ONP have equal measures.

60 ÷ 2 = 30

Thus, m∠MNO is 30 degrees.

*Method 2*

Let *x* represent the measure of ∠ONP.

Since ray PN bisects ∠MNO, then ∠ONP and ∠MNP have equal measures.

Thus, the measure of ∠MNP is also *x*

By the angle addition postulates:

m∠*ONP* + *m*∠*MNP* = *m*∠*MNO*

x + x = 60 *Recall that we let x be the measure of angles* ∠ONP *and* ∠*MNO*

2x = 60

2x/2 = 60/2 *Dividing both sides by 2*

x = 30

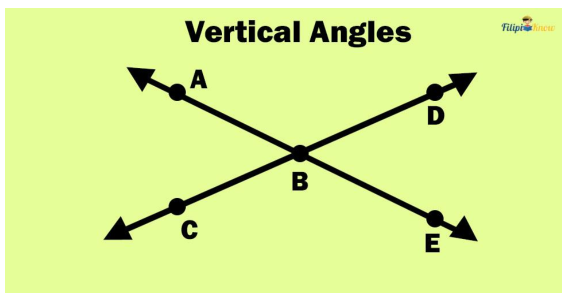
Since x represents the measure of ∠ONP, then m∠ONP = 30°.

## Angle Pairs

As the name suggests, an angle pair consists of two angles that are related in a certain way. In this section, let us discuss each of these angle pairs.

### 1. Vertical Angles

Vertical angles are formed when two lines intersect. Basically, vertical angles create two pairs of opposite rays




In the given figure above, ∠ABC and ∠DBE are vertical angles. Notice that this angle pair has two pairs of opposite rays, rays AB and BC (pair 1) and rays DB and BE (pair 2).

∠ABD and ∠CBE are also vertical angles (Can you identify the opposite pair of rays?).

However, ∠ABC and ∠*ABD* are not vertical angles since they do not form two pairs of opposite rays.

Here's an "informal" way to detect vertical angles quicker. Actually, you can imagine the vertical angles as the opposite openings of the letter "X". For instance, in the figure above, we can imagine the figure above as the letter X. Notice that angles ∠ABC and ∠DBE are opposite openings of this letter "X", so they are opposite angles. Same as with ∠ABD and ∠CBE, these angles are also opposite angles since they are opposite openings of the letter "X".

Sometimes, vertical angles are also called opposite angles.

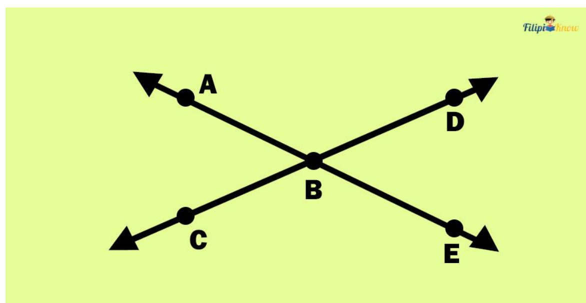
Here's an important property of vertical angles that you must always keep in mind:

**Theorem:**

*"Vertical angles formed by intersecting lines are always congruent."*

This means that any vertical angle will always have equal degree measurement.

Take a look again at our previous image:




∠ABC and ∠DBE are vertical angles. Using the vertical angle theorem, we can conclude that they are congruent. For instance, if the measure of ∠ABC is 45 degrees, then m∠DBE is also 45 degrees.

**Sample Problem:** Using the same figure above, if ∠CBE = 2x + 20 and ∠ABD = 120°. What is the value of x?

**Solution:** We know that angles ∠ABD and ∠CBE are vertical angles. Thus, they are congruent or have equal measures:

m∠ABD = m∠CBE

120 = 2x + 20

2x + 20 = 120 *Symmetric property*

2x = -20 + 120 *Transposition method*

2x = 100

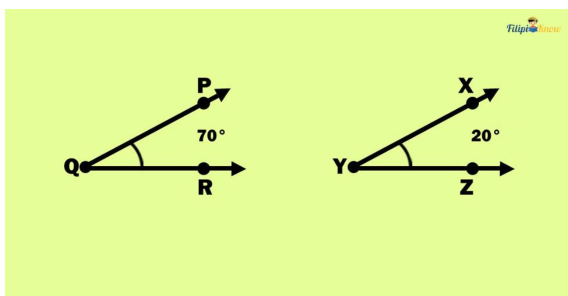
2x/2 = 100/2

x = 50

Thus, the value of *x* is 50.

### 2. Complementary Angles

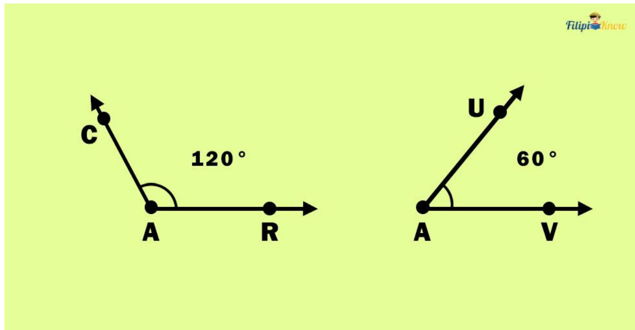
If the measurement of two angles has a sum equal to 90 degrees, then the angles are complementary angles.




∠PQR and ∠XYZ are complementary angles since the sum of their measurements is 90 degrees.

### 3. Supplementary Angles

Supplementary angles are almost similar to complementary angles except that the sum of their measures must be equal to 180 degrees. Thus, we can define supplementary angles as a pair of angles whose sum of measurements are 180 degrees.




In the image above, angles ∠CAR and ∠UAV are supplementary angles since the sum of their measurements is 180 degrees.

Let us solve some problems involving complementary and supplementary angles.

**Sample Problem 1:** An angle is a complement of another angle. If the measure of one of these angles is twice the measure of the other angle, what is the measure of the shorter angle?

**Solution:**

The problem does not provide us with any measurement of the angles. The only thing that we know is that they complement each other. Thus, we can state that the sum of these two angles is 90°.

First angle + Second angle = 90°.

The problem stated that the measure of one angle is twice the other, so it means that one angle is larger in measure than the other. To make our equation above more detailed:

Smaller angle + larger angle = 90°.

Let *x* be the measure of the smaller angle. The measure of the larger angle is twice (or two times) the smaller, so we let *2x* be the measure of the larger angle:

$x + 2x = 90$

We can now solve the value of *x* above:

$x + 2x = 90$

$3x = 90$ *Combining like terms*

$3x/3 = 90/3$ *Dividing both sides of the equation by 3*

$x = 30$

Since *x* represents the measure of the smaller angle, then **the smaller angle has a degree measure of 30 degrees.**

**Sample Problem 2:** Angles 1 and 2 are supplementary angles. Angle 1 was measured 60 degrees larger than twice the measure of angle 2. What is the measure of an angle that is complementary to angle 2?

**Solution:**

The first thing we have to do is to determine the measurements of angles 1 and 2.

It states that angles 1 and 2 are supplementary. Hence, the sum of their measurements must be 180:

angle 1 + angle 2 = 180

The measurement of angle 1 is 60 degrees larger than twice the measure of angle 2. Thus, angle 1 is larger than angle 2 (keep this in mind!)

Let *x* be the measure of angle 2.

Again, the measurement of angle 1 is 60 degrees larger than twice the measure of angle 2. We can express the measurement of the angle as *2x + 60* (*2x* is twice the measure of the first angle while the "plus 60 degrees" is for the "60 degrees larger" part of angle 1's description).

Again, *x* is the measurement of angle 2. *2x + 60* is the measurement of angle 1:

Going back to the earlier equation we established:

angle 1 + angle 2 = 180

x + (2x + 60) = 180

Let's solve for *x* in the equation we formed above:

x + (2x + 60) = 180

3x + 60 = 180 *Combining like terms*

3x = -60 + 180 *Transposition method*

3x = 120

3x⁄3 = 120⁄3 *Dividing both sides by 3*

x = 40

Now, since *x* represents the measure of angle 2, then angle 2 is 40° in measurement.

However, we are not done yet. The problem is not asking us to find the measurement of angle 2 but to find the measurement of its complement. The complement of angle 2 is just the angle such that when its measurement is added to the measure of angle 2, the result will be 90°.

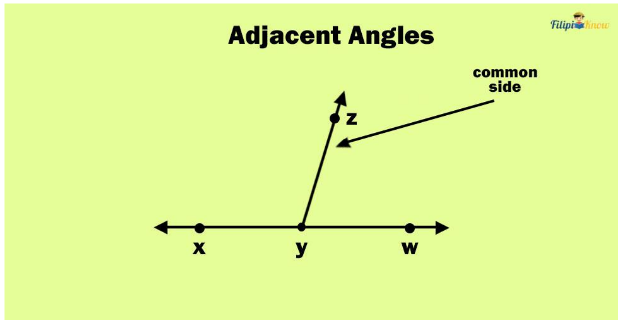
So, to find the measurement of the complement of angle 2, we just subtract 40° from 90°:

90° - 40° = 50°

So, the final answer for this problem is 50°.

## 4. Adjacent Angles and Linear Pairs

If two angles have a common vertex and a common side (or ray), then the angles are **adjacent angles**.

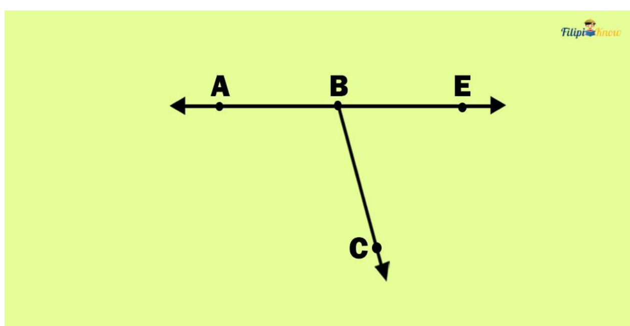



In the figure above, ∠XYZ and ∠WYZ share a common vertex which is point Y and a common side which is ray YZ. Thus, ∠XYZ and ∠WYZ are adjacent angles.

Now, if two adjacent angles are supplementary, then these angles are called **linear pairs.** Linear pairs will form a side which is a straight line.

In the previous figure, ∠XYZ and ∠WYZ form a side that is a straight line (line XW). Thus, ∠XYZ and ∠WYZ are linear pairs and they are supplementary.

**Sample Problem:** ∠ABC and ∠CBE are linear pairs. Determine the measure of ∠ABC if m∠CBE = 70 degrees.




Since ∠ABC and ∠CBE are linear pairs, then they are supplementary. We know that the measurements of supplementary angles have a sum of 180 degrees.

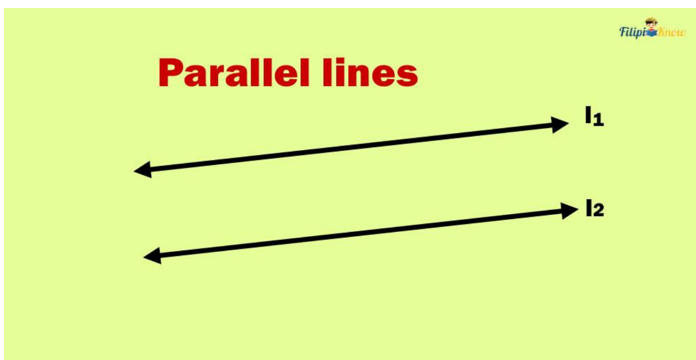
To find the measure of $\angle CBE$:

$m\angle ABC = 180 - 70 = 110$

Thus, the answer is 110 degrees.

## Angles Formed by Transversal Intersecting Parallel Lines

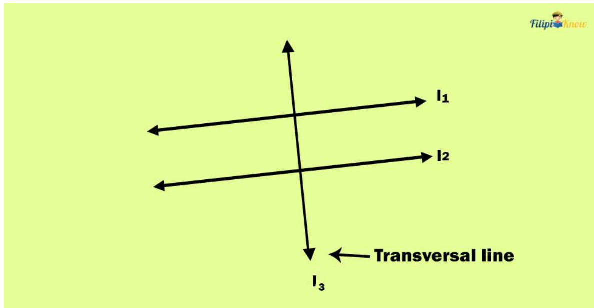
As a review, **parallel lines** are lines that do not meet or intersect.



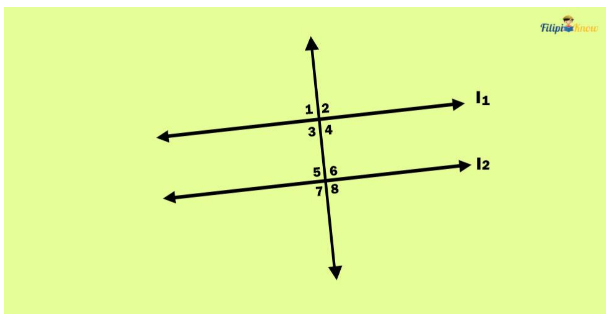

As you can see, lines $l_1$ and $l_2$ are parallel lines since they do not meet. Even if we extend the length of lines infinitely, it is certain that they will never intersect.

We use the symbol || to indicate that two lines are parallel. Hence, we can state "line $l_1$ and $l_2$ are parallel" in symbols as $l_1 \parallel l_2$

Now, if a line intersects two parallel lines (refer to the figure below), that line is called a **transversal line**.



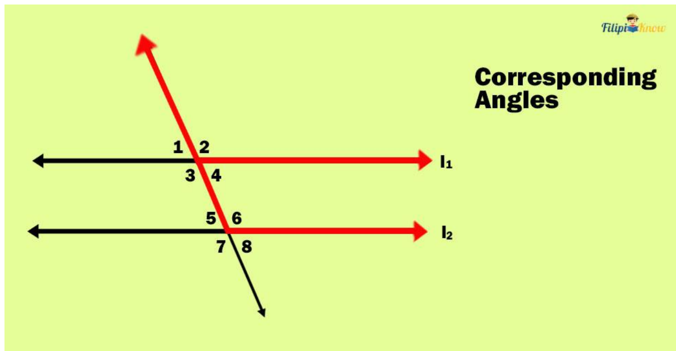

In the figure above, $l_3$ is a transversal line since it intersects two parallel lines $l_1$ and $l_2$.




When a transversal line intersects parallel lines, it is noticeable that various angles were formed. Throughout this reviewer, we will use numbers to identify the angles formed by transversal intersecting parallel lines. As you can see, 8 angles were formed, these angles are called **transversal angles**. In the next section, we will discuss how these transversal angles are related to each other.

## 1. Corresponding Angles

Corresponding angles are transversal angles that are on the same side of the transversal and have sides on the transversal going in the same direction and have their other sides going in parallel directions.




In the figure above, angles ∠2 and ∠6 are on the same side of the transversal and have sides on the transversal that is going in the same direction and have their other sides going in parallel directions. Likewise, ∠3 and ∠7 are corresponding angles because they exhibited the same properties.

On the other hand, ∠2 and ∠8 are not corresponding angles because although they are on the same side, their figures are not matching.

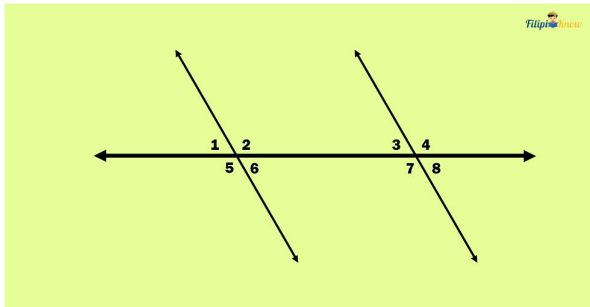
Here's an important theorem about corresponding angles.

**Theorem:**

> *"If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent."*

So, in the figure above, corresponding angles ∠2 and ∠6 have the same measurement. So, if m∠2 = 30°, then m∠6 should be 30° also.

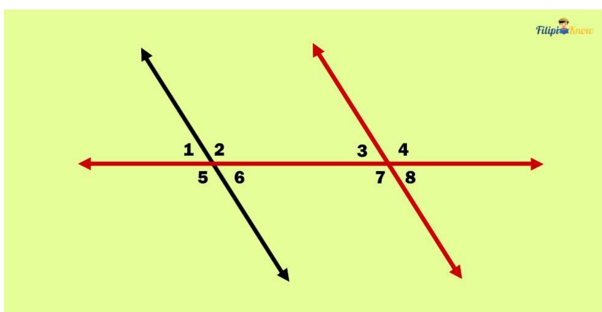
**Sample Problem 1:** In the figure below $l_1 \parallel l_2$, if m∠6 = 70°, determine m∠8.




**Solution**: In the figure above, angles 6 and 8 are corresponding angles since they are on the same "side" (below the transversal) and they have matching figures. Therefore, we can state that angles 6 and 8 are congruent. If m∠6 = 70°, then m∠8 = 70°.

**Sample Problem 2:** Using the same figure in the previous example, if m∠8 = 70°, determine m∠3.

**Solution**: If you take a look at the figure again, you will notice that angles ∠3 and ∠8 are vertical angles since they create two pairs of opposite rays. We know that vertical angles are congruent, so if m∠8 = 70°, then m∠3 = 70°




**Sample Problem 3:** Using the same figure in the previous example, if m∠8 = 70°, determine m∠7.

**Solution:** Notice that angles 7 and angle 8 are linear pairs since they share a common side and their sides form a straight line. From our previous discussion about linear pairs, we have learned that linear pairs are supplementary. So, angles 8 and 7 are supplementary angles:

Supplementary angles have a sum of measures of 180 degrees:

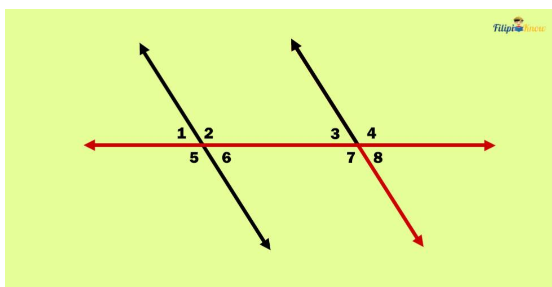
$m\angle 7 + m\angle 8 = 180$

We know that $m\angle 8$ is 70 degrees:

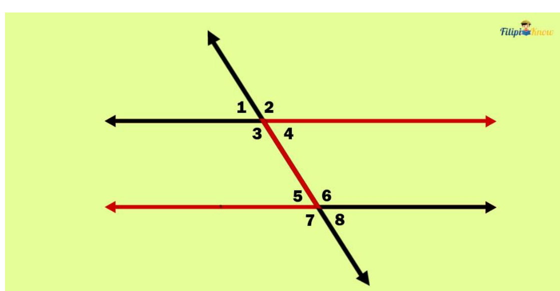
$m\angle 7 + 70 = 180$

Solving for m $m\angle 7$:

$m\angle 7 = 180 - 70 = 110°$




## 2. Alternate Interior Angles

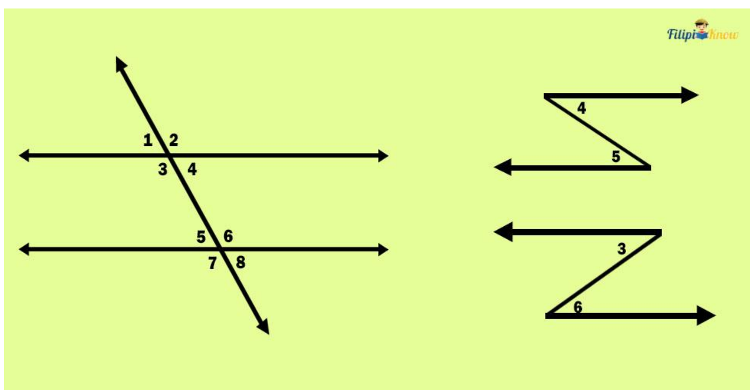


Alternate interior angles are transversal angles that are in the inner portion of the parallel lines but are on the opposite side of the transversal.

In the figure above, angles 4 and 5 are alternate interior angles since they are in the interior portion of the parallel lines and they are on the opposite sides of the transversal (angle 4 is on the right side, angle 5 is on the left side).

Angles 3 and 6 are also alternate interior angles.

If you have noticed, alternate interior angles formed this weird letter “S” shape. If you look again at angles 4 and 5, it seems that they formed a letter “S”-like figure. Angles 3 and 6 also formed the inverse of this letter “S”-like figure. Look at the figure below to visualize better what alternate interior angles look like.




Here’s an important theorem about alternate interior angles:

**Theorem**:

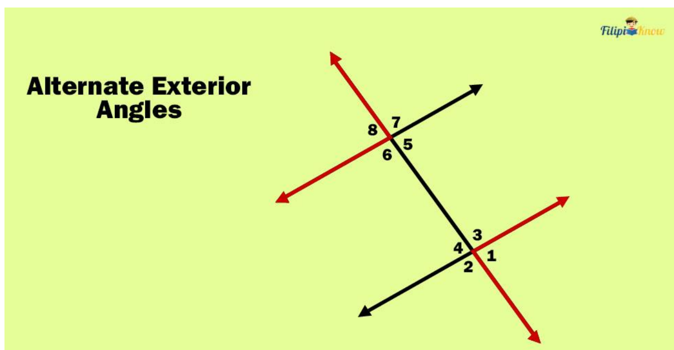
> *“If two parallel lines are cut by a transversal, then the alternate interior angles formed are congruent.”*

The theorem above tells us that alternate interior angles have the same measurement.

Looking at the given figure above, we can state that angles 4 and 5 are congruent since they are alternate interior angles. Moreover, we can also state that angles 3 and 6 are congruent since they are also alternate interior angles.

## 3. Alternate Exterior Angles

Alternate exterior angles are the opposite of the alternate interior angles. Alternate exterior angles are a pair of angles that are in the exterior portion of the parallel lines and are on the opposite sides of the transversal line.




In the figure above, ∠1 and ∠8 are alternate exterior angles since they are both on the exterior of the parallel lines and they are also on the opposite sides of the transversal (∠1 is on the right side of the transversal, ∠8 is on the left side).

There's an important theorem regarding alternate exterior angles. It is stated below.

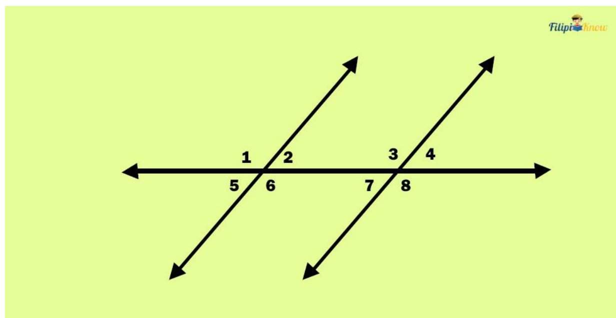
**Theorem:**

> *"If two parallel lines are cut by a transversal, then the alternate exterior angles formed are congruent."*

The theorem above tells us that if two angles are alternate exterior angles, then these angles have an equal measurement or are congruent.

Hence, in the figure we have above, we can conclude using the theorem that angles 1 and 8 are congruent or have the same measurement since they are alternate exterior angles.

**Sample Problem:** Using the given figure below, determine the measures of angles ∠2, ∠3, ∠6, and ∠8 if m∠1 = 80°




**Solution:**

Let us start determining the degree measure of ∠2. If you look at the given figure above, you will notice that angles ∠1 and ∠2 are linear pairs since they share a common side and their remaining sides form a straight line. We know that linear pairs are supplements of each other, so the sum of measures of ∠1 and ∠2 should be 180°:

m∠1 + m∠2 = 180°

It is given that m∠1 = 80°, so let's plug it into the equation above:

80 + m∠2 = 180°

m∠2 = 180 - 80

m∠2 = 100°

As we can see, the computed measure of ∠2 is 100° or **m∠2 = 100°.**

Now, let us determine the measure of ∠3. Take a look again at the given figure. *What can you say about ∠1 and ∠3?* Yes, they are corresponding angles.

As per the previous theorem we have discussed, corresponding angles are congruent. Since ∠1 and ∠3 are congruent, then these angles have the same measurement. So, if m∠1 = 80°**,** then m∠3 should be equal to 80° also. Thus, **m∠3 = 80°**.

This time, let us determine the measure of ∠6. *Which angle do you think we can use to determine the measure of ∠6??* Well, you can use either angle 1 or angle 2.

If you use ∠1, then ∠1 and ∠6 are vertical angles. Since vertical angles are congruent (as per the vertical angle theorem), if m∠1 = 80°, then **m∠6 = 80°**.

On the other hand, if you use ∠2 instead, ∠2 and ∠6 are linear pairs. Since linear pairs are supplements of each other, then the sum of their measurement is 180°**.** We have computed earlier that m∠2 = 100°, so to find the measure of ∠6:

m∠6 = 180 - m∠2

m∠6 = 180 - 100

m∠6 = 80°

Hence, m∠6 = 80°

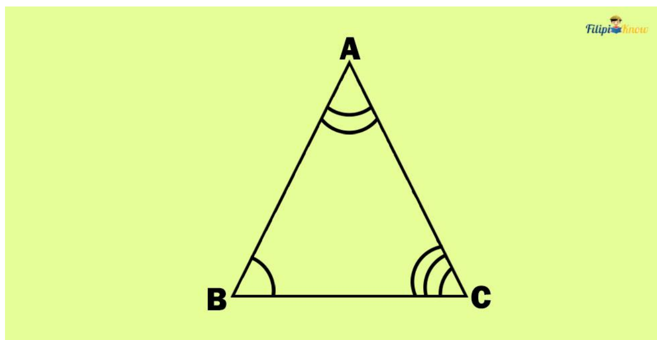
Note that whether you use angle 1 or 2, you can still derive the same measurement for angle 6.

Lastly, to find the measure of ∠8, we can use the measurement of ∠1. ∠1 and ∠8 are alternate exterior angles since they are both located in the exterior portion of the parallel lines and are on the opposite sides of the transversal line (look at the given figure above). We know that alternate exterior angles are congruent based on a previous theorem. Hence, if m∠1 = 80°, then **m∠8 = 80°**.

‘

## Interior Angles of a Polygon

Angles that are located inside a polygon are called **interior angles.** As you may recall, a polygon is a plane figure that is composed of sides and vertices where these sides meet. Take a look at the triangle ABC (or △ABC) below:




The triangle above has three interior angles namely ∠ABC, ∠ACB, and ∠BAC. We put arcs in the triangle to indicate these interior angles.

*Did you know that if you draw any kind of triangle, the total measurement of its interior angles will always be 180°?*

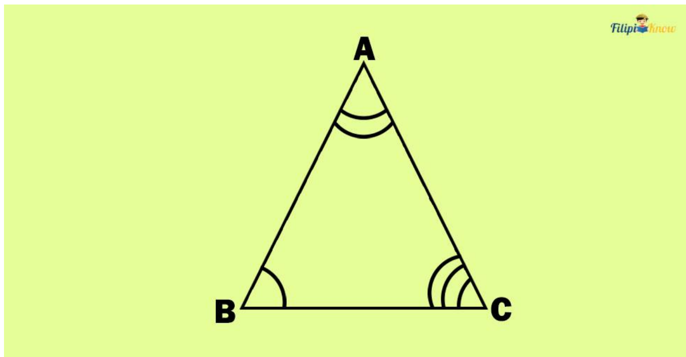
Yes, it is true that the sum of the interior angles of any triangle is 180°. We state this concept formally in the theorem below:

**Triangle Sum Theorem:**

> *"The sum of the measurements of all the interior angles of any triangle is 180°."*

So, whether you, your friend, or a stranger draws a triangle, the sum of the interior angles of that triangle will always be exactly 180°.

**Sample Problem:** If m∠ABC = 3x + 15, m∠ACB = x + 20, and m∠BAC = x, determine the value of x (refer to the figure below).




**Solution:**

Since the given angles are interior angles of the triangle above, then we are sure that the sum of the measurements of these angles is 180° because of the triangle sum theorem.

$m\angle ABC + m\angle ACB + m\angle BAC = 180°$

$(3x + 15) + (x + 20) + x = 180°$ *Input the given values in the problem*

$5x + 35 = 180°$ *Combining like terms*

$5x = -35 + 180$ *Transposition method*

$5x = 145$

$5x/5 = 145/5$ *Dividing both sides of the equation by 5*

$x = 29$

Thus, the value of x is 29.

## General Formula for the Sum of Interior Angles of a Polygon

A polygon with *n* sides has *n* interior angles. So, if a triangle has 3 sides, then it has 3 interior angles also. Meanwhile, a square has 4 sides, so it has 4 interior angles as well. A pentagon has five sides, so it has five interior angles also.

We learned in the previous section that the sum of the interior angles of a triangle is always 180°. *How about quadrilaterals, pentagons, hexagons, or decagons? How do we find the sum of their interior angles?*

There is a general formula we can use to determine the sum of the interior angles of a polygon with *n* sides. This formula is presented below:

The sum of the interior angles of a polygon with n sides is given by the formula:

**Sum of interior angles = 180(n - 2)**

So, a quadrilateral that has *n = 4* sides has a sum of interior angles:

*Sum of interior angles = 180(4 - 2) = 180(2) = 360°*

Any quadrilateral (four-sided polygon) such as square, rectangle, parallelogram, trapezoid, etc. will always have a sum of measurements of interior angles equal to 360°

**Sample Problem:** A polygon has 12 sides (i.e., dodecagon). What is the sum of its interior angles?

**Solution:**

Using our formula and n = 12:

Sum of interior angles = 180(n - 2)

Sum of interior angles = 180(12 - 2)

Sum of interior angles = 180(10) = 1800°

Hence, the sum of the measurements of the interior angles of a dodecagon is 1800°.

## Measure of an Interior Angle of a Regular Polygon

As a consequence of the formula above, if a polygon is a regular polygon (which means that all of its sides and angles are congruent), then the measurement of an interior angle of a regular polygon with *n* sides can be computed as:

$$\text{Measurement of an interior angle of a regular polygon} = \frac{180(n-2)}{n}$$

Suppose an equilateral and equiangular triangle where all of its sides and angles are congruent. Now, the measure of one of its angles can be calculated using the formula above.

Using n = 3:

Measurement of an interior angle of an equilateral and equiangular triangle

$$\frac{180(3-2)}{3} = \frac{180(1)}{3} = \frac{180}{3} = 60$$

Thus, the measure of an interior angle of an equilateral and equiangular triangle is 60°.

**Sample Problem:** A regular polygon has n = 10 sides (decagon). Determine the measurement of one of its interior angles.

**Solution:**

Using the formula for the measurement of an interior angle of a regular polygon:

Measurement of an interior angle of a regular polygon = $\frac{180(n-2)}{n}$

Using n = 10:

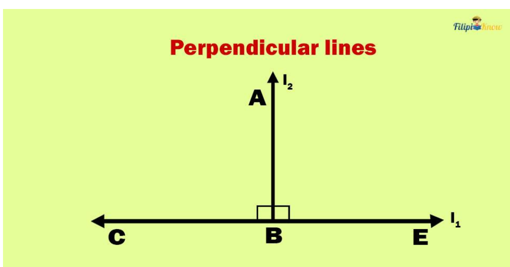
$$\frac{180(n-2)}{n} = \frac{180(10-2)}{10} = \frac{180(8)}{10} = 144$$

Thus, the measure of an interior angle of a regular polygon with 10 sides (decagon) is 144°.

## Perpendicularity

If two lines intersect and these lines form right angles, then these lines are **perpendicular**. In other words, perpendicular lines form right angles.

An informal way to detect perpendicular lines is by looking at the “T shape” formed by these lines. Notice that perpendicular lines form a letter “T” or an inverted letter “T.”

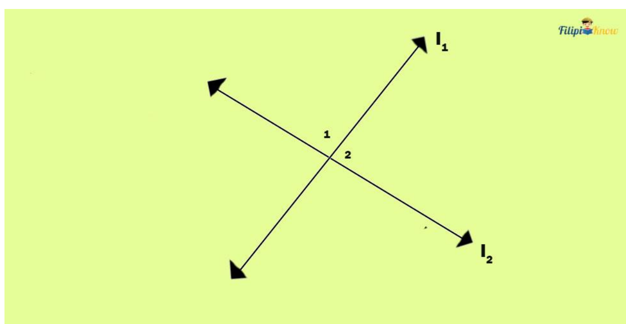



*Perpendicular lines form 90-degree angles*

In the figure above, lines $l_1$ and $l_2$ are perpendicular. We use the symbol to indicate that two lines are perpendicular. Hence, $l_1$ $l_2$. Since these lines are perpendicular, then ∠ABC and ∠ABE are right angles with m∠ABC and m∠ABE both equal to 90 degrees.

In addition to this, as you can see above, the angles formed by perpendicular lines are also linear pairs since they share a common side (in the figure above, ray AB) and their remaining sides form a straight line,

**Sample Problem**: Lines $l_1$ and $l_2$ are perpendicular. If angle 1 measures x + 5 degrees. What is the value of *x*?




**Solution:** Angles formed by perpendicular lines always have a measurement of 90 degrees. Thus, angle 1 should have a degree measurement of 90. Hence, to find the value of x:

x + 5 = 90

x = -5 + 90

x = 85

The answer is 85.